Future Pre-Calculus students:

This packet contains topics that you must know in order to be successful in Pre-Calculus A, B, and C. It is the first three sections of the textbook, which will give you a good indication of what will be expected of you and of what your workload in this class will be. Note the format of the sections, the explanations, the Examples, and the type of homework Exercises. Knowing this material will be essential, as it will be the foundation of the course.

Taking all three of these courses is highly recommended if you are interested in taking AP Calculus, and especially AP Calculus BC. And while Pre-Calculus C is not a prerequisite for AP Calculus, it covers topics such as sequences, derivatives, limits, and integrals, which are a focus in AP Calculus. So when you are signing up for classes for next year, choose all three trimesters of Pre-Calculus.

Below is an assignment. It will not be checked. However, in order to assess your knowledge and in order to gauge your dedication to this course, you will be given a practice quiz over the assigned material on the third day of Pre-Calculus A. An actual quiz over the assigned material, for which a grade <u>will</u> be recorded, will be on the fourth day. You will <u>not</u> be allowed to use a calculator on this quiz or for most of the course, so do not allow yourself to use one on the assigned problems. The answers are also in this packet so that you may check your work.

Note that this assignment covers <u>review material only</u>. In other words, you should have already learned it in previous courses. You will, however, be given a short amount of time to ask questions regarding the assignment during the first two days of Pre-Calculus A.

I look forward to working with you. If you have any questions, feel free to email me.

Sincerely,
Ms. Westen (room 318)

<u>Assignment</u>

- ❑ Read page xviii.
- ❑ Read page 1.

- ❑ Read and study section 1.1.
- ❑ Do homework Exercises #1, 13, 18, 19, 22, 28, 31-33, 40, 41, 44, 45, 48, 55-59 (odds), 63, 64, 70, 76, 87.

- ❑ Read and study section 1.2 (through and including Example 7).
- ❑ Do homework Exercises #1, 40, 44, 83, 86, 88, 95, 100.
- ❑ Note that without a calculator, you would only be expected to round the answer to Exercise #100 to ≈ $50,000.

- ❑ Read and study section 1.3 (through and including Example 3).
- ❑ Do homework Exercises #13, 17, 18, 20, 24, 27, 29, 49, 55, 60.

- ❑ Check and correct answers to each of the homework Exercises listed above.

TO THE STUDENT

This textbook was written for you to use as a guide to mastering precalculus mathematics. Here are some suggestions to help you get the most out of your course.

First of all, you should read the appropriate section of text *before* you attempt your homework problems. Reading a mathematics text is quite different from reading a novel, a newspaper, or even another textbook. You may find that you have to reread a passage several times before you understand it. Pay special attention to the examples, and work them out yourself with pencil and paper as you read. Then do the linked exercises referred to in *"Now Try Exercise . . ."* at the end of each example. With this kind of preparation you will be able to do your homework much more quickly and with more understanding.

Don't make the mistake of trying to memorize every single rule or fact you may come across. Mathematics doesn't consist simply of memorization. Mathematics is a *problem-solving art*, not just a collection of facts. To master the subject you must solve problems—lots of problems. Do as many of the exercises as you can. Be sure to write your solutions in a logical, step-by-step fashion. Don't give up on a problem if you can't solve it right away. Try to understand the problem more clearly—reread it thoughtfully and relate it to what you have learned from your teacher and from the examples in the text. Struggle with it until you solve it. Once you have done this a few times you will begin to understand what mathematics is really all about.

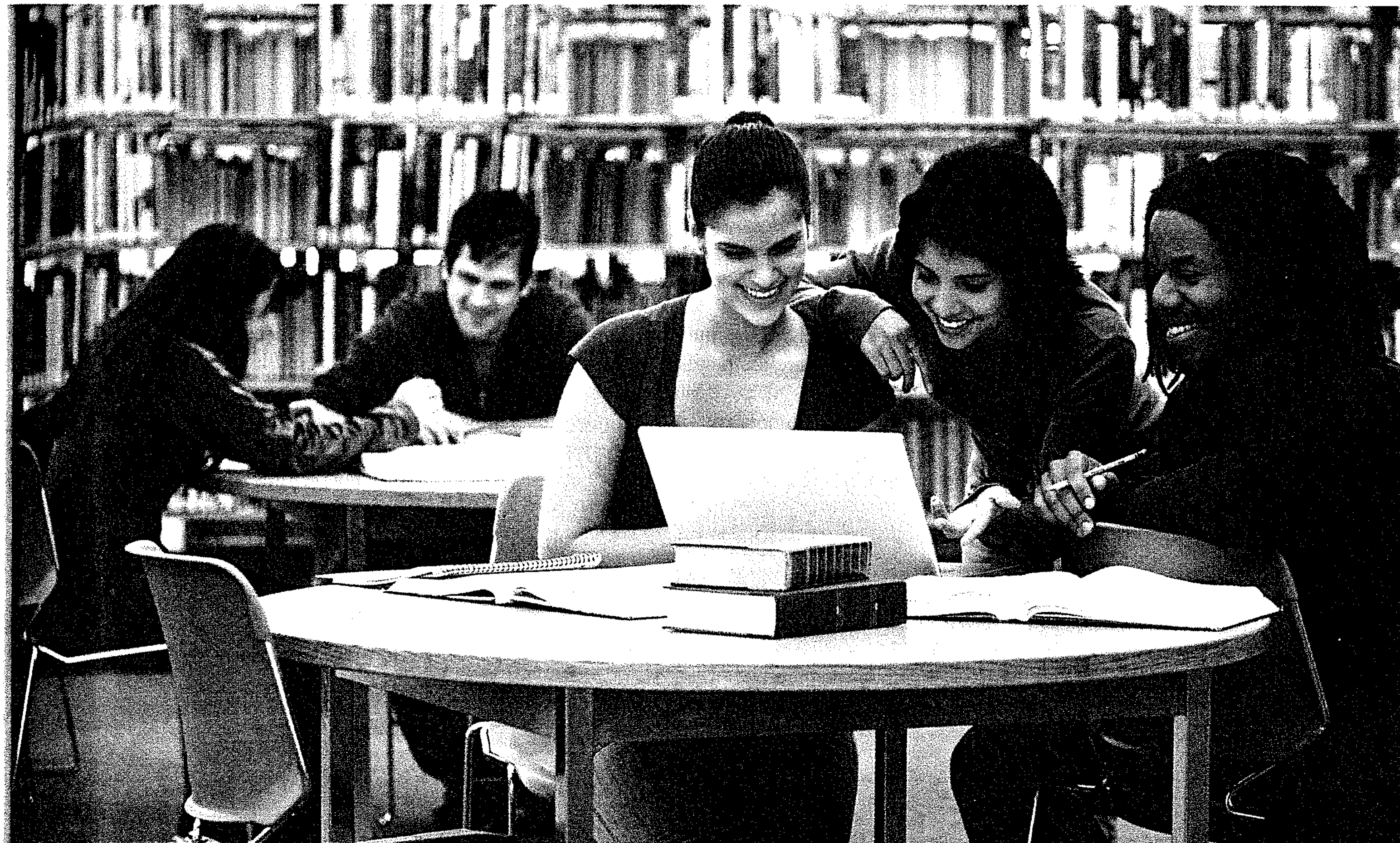
Answers to the odd-numbered exercises, as well as all the answers (even and odd) to the concept exercises and chapter tests, appear at the back of the book. If your answer differs from the one given, don't immediately assume that you are wrong. There may be a calculation that connects the two answers and makes both correct. For example, if you get $1/(\sqrt{2} - 1)$ but the answer given is $1 + \sqrt{2}$, your answer *is* correct, because you can multiply both numerator and denominator of your answer by $\sqrt{2} + 1$ to change it to the given answer. In rounding approximate answers, follow the guidelines in Appendix B: *Calculations and Significant Figures.*

The symbol ⊘ is used to warn against committing an error. We have placed this symbol in the margin to point out situations where we have found that many of our students make the same mistake.

Abbreviations

The following abbreviations are used throughout the text.

cm	centimeter	**kPa**	kilopascal	**N**	Newton
dB	decibel	**L**	liter	**qt**	quart
F	farad	**lb**	pound	**oz**	ounce
ft	foot	**lm**	lumen	**s**	second
g	gram	**M**	mole of solute per liter of solution	**Ω**	ohm
gal	gallon			**V**	volt
h	hour			**W**	watt
H	henry	**m**	meter	**yd**	yard
Hz	Hertz	**mg**	milligram	**yr**	year
in.	inch	**MHz**	megahertz	**°C**	degree Celsius
J	Joule	**mi**	mile	**°F**	degree Fahrenheit
kcal	kilocalorie	**min**	minute	**K**	Kelvin
kg	kilogram	**mL**	milliliter	$\Rightarrow$	implies
km	kilometer	**mm**	millimeter	$\Leftrightarrow$	is equivalent to



Blend Images/Alamy

1 Fundamentals

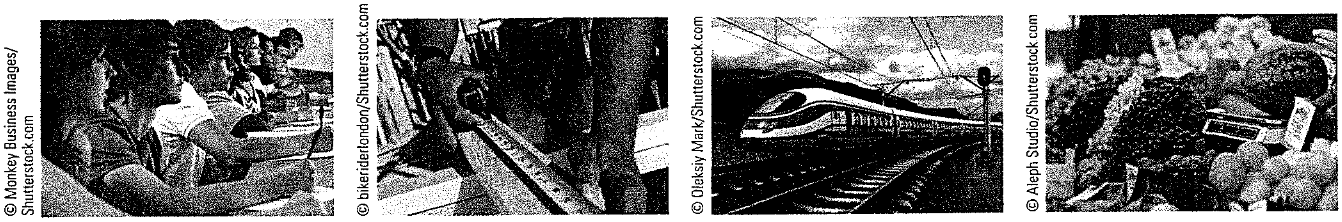
In this first chapter we review the real numbers, equations, and the coordinate plane. You are probably already familiar with these concepts, but it is helpful to get a fresh look at how these ideas work together to solve problems and model (or describe) real-world situations.

In the *Focus on Modeling* at the end of the chapter we learn how to find linear trends in data and how to use these trends to make predictions about the future.

1.1 REAL NUMBERS

■ Real Numbers ■ Properties of Real Numbers ■ Addition and Subtraction ■ Multiplication and Division ■ The Real Line ■ Sets and Intervals ■ Absolute Value and Distance

In the real world we use numbers to measure and compare different quantities. For example, we measure temperature, length, height, weight, blood pressure, distance, speed, acceleration, energy, force, angles, age, cost, and so on. Figure 1 illustrates some situations in which numbers are used. Numbers also allow us to express relationships between different quantities—for example, relationships between the radius and volume of a ball, between miles driven and gas used, or between education level and starting salary.



Count Length Speed Weight

FIGURE 1 Measuring with real numbers

■ Real Numbers

Let's review the types of numbers that make up the real number system. We start with the **natural numbers**:

$$1, 2, 3, 4, \ldots$$

The different types of real numbers were invented to meet specific needs. For example, natural numbers are needed for counting, negative numbers for describing debt or below-zero temperatures, rational numbers for concepts like "half a gallon of milk," and irrational numbers for measuring certain distances, like the diagonal of a square.

The **integers** consist of the natural numbers together with their negatives and 0:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$$

We construct the **rational numbers** by taking ratios of integers. Thus any rational number r can be expressed as

$$r = \frac{m}{n}$$

where m and n are integers and $n \neq 0$. Examples are

$$\tfrac{1}{2} \qquad -\tfrac{3}{7} \qquad 46 = \tfrac{46}{1} \qquad 0.17 = \tfrac{17}{100}$$

(Recall that division by 0 is always ruled out, so expressions like $\frac{3}{0}$ and $\frac{0}{0}$ are undefined.) There are also real numbers, such as $\sqrt{2}$, that cannot be expressed as a ratio of integers and are therefore called **irrational numbers**. It can be shown, with varying degrees of difficulty, that these numbers are also irrational:

$$\sqrt{3} \qquad \sqrt{5} \qquad \sqrt[3]{2} \qquad \pi \qquad \frac{3}{\pi^2}$$

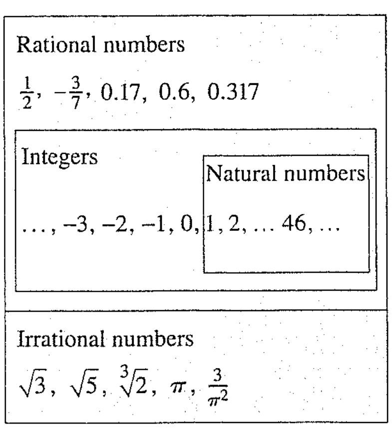


FIGURE 2 The real number system

The set of all real numbers is usually denoted by the symbol $\mathbb{R}$. When we use the word *number* without qualification, we will mean "real number." Figure 2 is a diagram of the types of real numbers that we work with in this book.

Every real number has a decimal representation. If the number is rational, then its corresponding decimal is repeating. For example,

$$\tfrac{1}{2} = 0.5000\ldots = 0.5\overline{0} \qquad \tfrac{2}{3} = 0.66666\ldots = 0.\overline{6}$$

$$\tfrac{157}{495} = 0.3171717\ldots = 0.3\overline{17} \qquad \tfrac{9}{7} = 1.285714285714\ldots = 1.\overline{285714}$$

A repeating decimal such as

$$x = 3.5474747\ldots$$

is a rational number. To convert it to a ratio of two integers, we write

$$\begin{aligned} 1000x &= 3547.47474747\ldots \\ 10x &= 35.47474747\ldots \\ \hline 990x &= 3512.0 \end{aligned}$$

Thus $x = \frac{3512}{990}$. (The idea is to multiply x by appropriate powers of 10 and then subtract to eliminate the repeating part.)

(The bar indicates that the sequence of digits repeats forever.) If the number is irrational, the decimal representation is nonrepeating:

$$\sqrt{2} = 1.414213562373095\ldots \qquad \pi = 3.141592653589793\ldots$$

If we stop the decimal expansion of any number at a certain place, we get an approximation to the number. For instance, we can write

$$\pi \approx 3.14159265$$

where the symbol $\approx$ is read "is approximately equal to." The more decimal places we retain, the better our approximation.

Properties of Real Numbers

We all know that $2 + 3 = 3 + 2$, and $5 + 7 = 7 + 5$, and $513 + 87 = 87 + 513$, and so on. In algebra we express all these (infinitely many) facts by writing

$$a + b = b + a$$

where a and b stand for any two numbers. In other words, "$a + b = b + a$" is a concise way of saying that "when we add two numbers, the order of addition doesn't matter." This fact is called the *Commutative Property* of addition. From our experience with numbers we know that the properties in the following box are also valid.

PROPERTIES OF REAL NUMBERS

Property	Example	Description
Commutative Properties		
$a + b = b + a$	$7 + 3 = 3 + 7$	When we add two numbers, order doesn't matter.
$ab = ba$	$3 \cdot 5 = 5 \cdot 3$	When we multiply two numbers, order doesn't matter.
Associative Properties		
$(a + b) + c = a + (b + c)$	$(2 + 4) + 7 = 2 + (4 + 7)$	When we add three numbers, it doesn't matter which two we add first.
$(ab)c = a(bc)$	$(3 \cdot 7) \cdot 5 = 3 \cdot (7 \cdot 5)$	When we multiply three numbers, it doesn't matter which two we multiply first.
Distributive Property		
$a(b + c) = ab + ac$ $(b + c)a = ab + ac$	$2 \cdot (3 + 5) = 2 \cdot 3 + 2 \cdot 5$ $(3 + 5) \cdot 2 = 2 \cdot 3 + 2 \cdot 5$	When we multiply a number by a sum of two numbers, we get the same result as we get if we multiply the number by each of the terms and then add the results.

The Distributive Property applies whenever we multiply a number by a sum. Figure 3 explains why this property works for the case in which all the numbers are positive integers, but the property is true for any real numbers a, b, and c.

The Distributive Property is crucial because it describes the way addition and multiplication interact with each other.

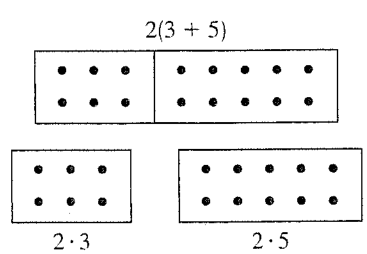


FIGURE 3 The Distributive Property

EXAMPLE 1 Using the Distributive Property

(a) $2(x + 3) = 2 \cdot x + 2 \cdot 3$ — Distributive Property

$= 2x + 6$ — Simplify

(b) $(a + b)(x + y) = (a + b)x + (a + b)y$ — Distributive Property

$= (ax + bx) + (ay + by)$ — Distributive Property

$= ax + bx + ay + by$ — Associative Property of Addition

In the last step we removed the parentheses because, according to the Associative Property, the order of addition doesn't matter.

Now Try Exercise 15

Addition and Subtraction

The number 0 is special for addition; it is called the **additive identity** because $a + 0 = a$ for any real number a. Every real number a has a **negative**, $-a$, that satisfies $a + (-a) = 0$. **Subtraction** is the operation that undoes addition; to subtract a number from another, we simply add the negative of that number. By definition

$$a - b = a + (-b)$$

To combine real numbers involving negatives, we use the following properties.

Don't assume that $-a$ is a negative number. Whether $-a$ is negative or positive depends on the value of a. For example, if $a = 5$, then $-a = -5$, a negative number, but if $a = -5$, then $-a = -(-5) = 5$ (Property 2), a positive number.

PROPERTIES OF NEGATIVES

Property	Example
1. $(-1)a = -a$	$(-1)5 = -5$
2. $-(-a) = a$	$-(-5) = 5$
3. $(-a)b = a(-b) = -(ab)$	$(-5)7 = 5(-7) = -(5 \cdot 7)$
4. $(-a)(-b) = ab$	$(-4)(-3) = 4 \cdot 3$
5. $-(a + b) = -a - b$	$-(3 + 5) = -3 - 5$
6. $-(a - b) = b - a$	$-(5 - 8) = 8 - 5$

Property 6 states the intuitive fact that $a - b$ and $b - a$ are negatives of each other. Property 5 is often used with more than two terms:

$$-(a + b + c) = -a - b - c$$

EXAMPLE 2 Using Properties of Negatives

Let x, y, and z be real numbers.

(a) $-(x + 2) = -x - 2$ — Property 5: $-(a + b) = -a - b$

(b) $-(x + y - z) = -x - y - (-z)$ — Property 5: $-(a + b) = -a - b$

$= -x - y + z$ — Property 2: $-(-a) = a$

Now Try Exercise 23

Multiplication and Division

The number 1 is special for multiplication; it is called the **multiplicative identity** because $a \cdot 1 = a$ for any real number a. Every nonzero real number a has an **inverse**, $1/a$, that satisfies $a \cdot (1/a) = 1$. **Division** is the operation that undoes multiplication; to divide by a number, we multiply by the inverse of that number. If $b \neq 0$, then, by definition,

$$a \div b = a \cdot \frac{1}{b}$$

We write $a \cdot (1/b)$ as simply a/b. We refer to a/b as the **quotient** of a and b or as the **fraction** a over b; a is the **numerator** and b is the **denominator** (or **divisor**). To combine real numbers using the operation of division, we use the following properties.

PROPERTIES OF FRACTIONS

Property	Example	Description
1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$	When **multiplying fractions**, multiply numerators and denominators.
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$	When **dividing fractions**, invert the divisor and multiply.
3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$	When **adding fractions** with the **same denominator**, add the numerators.
4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{2}{5} + \frac{3}{7} = \frac{2 \cdot 7 + 3 \cdot 5}{35} = \frac{29}{35}$	When **adding fractions** with **different denominators**, find a common denominator. Then add the numerators.
5. $\frac{ac}{bc} = \frac{a}{b}$	$\frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$	**Cancel** numbers that are **common factors** in numerator and denominator.
6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$	$\frac{2}{3} = \frac{6}{9}$, so $2 \cdot 9 = 3 \cdot 6$	**Cross-multiply**.

When adding fractions with different denominators, we don't usually use Property 4. Instead we rewrite the fractions so that they have the smallest possible common denominator (often smaller than the product of the denominators), and then we use Property 3. This denominator is the Least Common Denominator (LCD) described in the next example.

EXAMPLE 3 Using the LCD to Add Fractions

Evaluate: $\frac{5}{36} + \frac{7}{120}$

SOLUTION Factoring each denominator into prime factors gives

$$36 = 2^2 \cdot 3^2 \qquad \text{and} \qquad 120 = 2^3 \cdot 3 \cdot 5$$

We find the least common denominator (LCD) by forming the product of all the prime factors that occur in these factorizations, using the highest power of each prime factor. Thus the LCD is $2^3 \cdot 3^2 \cdot 5 = 360$. So

$$\frac{5}{36} + \frac{7}{120} = \frac{5 \cdot 10}{36 \cdot 10} + \frac{7 \cdot 3}{120 \cdot 3} \qquad \text{Use common denominator}$$

$$= \frac{50}{360} + \frac{21}{360} = \frac{71}{360} \qquad \text{Property 3: Adding fractions with the same denominator}$$

Now Try Exercise 29

The Real Line

The real numbers can be represented by points on a line, as shown in Figure 4. The positive direction (toward the right) is indicated by an arrow. We choose an arbitrary reference point O, called the **origin**, which corresponds to the real number 0. Given any convenient unit of measurement, each positive number x is represented by the point on the line a distance of x units to the right of the origin, and each negative number $-x$ is represented by the point x units to the left of the origin. The number associated with the point P is called the coordinate of P, and the line is then called a **coordinate line**, or a **real number line**, or simply a **real line**. Often we identify the point with its coordinate and think of a number as being a point on the real line.

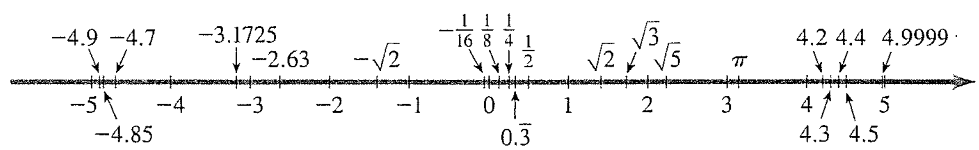


FIGURE 4 The real line

The real numbers are *ordered*. We say that ***a* is less than *b*** and write $a < b$ if $b - a$ is a positive number. Geometrically, this means that a lies to the left of b on the number line. Equivalently, we can say that ***b* is greater than *a*** and write $b > a$. The symbol $a \le b$ (or $b \ge a$) means that either $a < b$ or $a = b$ and is read "a is less than or equal to b." For instance, the following are true inequalities (see Figure 5):

$$7 < 7.4 < 7.5 \qquad -\pi < -3 \qquad \sqrt{2} < 2 \qquad 2 \le 2$$

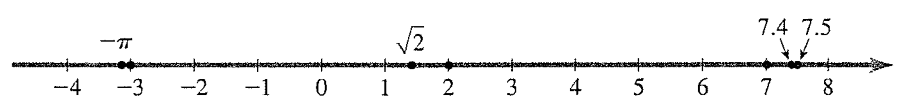


FIGURE 5

Sets and Intervals

A **set** is a collection of objects, and these objects are called the **elements** of the set. If S is a set, the notation $a \in S$ means that a is an element of S, and $b \notin S$ means that b is not an element of S. For example, if Z represents the set of integers, then $-3 \in Z$ but $\pi \notin Z$.

Some sets can be described by listing their elements within braces. For instance, the set A that consists of all positive integers less than 7 can be written as

$$A = \{1, 2, 3, 4, 5, 6\}$$

We could also write A in **set-builder notation** as

$$A = \{x \mid x \text{ is an integer and } 0 < x < 7\}$$

which is read "A is the set of all x such that x is an integer and $0 < x < 7$."

DISCOVERY PROJECT

Real Numbers in the Real World

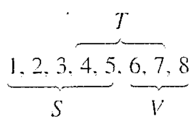
Real-world measurements always involve units. For example, we usually measure distance in feet, miles, centimeters, or kilometers. Some measurements involve different types of units. For example, speed is measured in miles per hour or meters per second. We often need to convert a measurement from one type of unit to another. In this project we explore different types of units used for different purposes and how to convert from one type of unit to another. You can find the project at **www.stewartmath.com**.

If S and T are sets, then their **union** $S \cup T$ is the set that consists of all elements that are in S *or* T (or in both). The **intersection** of S and T is the set $S \cap T$ consisting of all elements that are in both S *and* T. In other words, $S \cap T$ is the common part of S and T. The **empty set**, denoted by $\varnothing$, is the set that contains no element.

EXAMPLE 4 Union and Intersection of Sets

If $S = \{1, 2, 3, 4, 5\}$, $T = \{4, 5, 6, 7\}$, and $V = \{6, 7, 8\}$, find the sets $S \cup T$, $S \cap T$, and $S \cap V$.

SOLUTION



$S \cup T = \{1, 2, 3, 4, 5, 6, 7\}$ — All elements in S or T

$S \cap T = \{4, 5\}$ — Elements common to both S and T

$S \cap V = \varnothing$ — S and V have no element in common

Now Try Exercise 41

Certain sets of real numbers, called **intervals**, occur frequently in calculus and correspond geometrically to line segments. If $a < b$, then the **open interval** from a to b consists of all numbers between a and b and is denoted (a, b). The **closed interval** from a to b includes the endpoints and is denoted $[a, b]$. Using set-builder notation, we can write

$$(a, b) = \{x \mid a < x < b\} \qquad [a, b] = \{x \mid a \le x \le b\}$$

FIGURE 6 The open interval (a, b)

FIGURE 7 The closed interval $[a, b]$

Note that parentheses ( ) in the interval notation and open circles on the graph in Figure 6 indicate that endpoints are *excluded* from the interval, whereas square brackets [ ] and solid circles in Figure 7 indicate that the endpoints are *included*. Intervals may also include one endpoint but not the other, or they may extend infinitely far in one direction or both. The following table lists the possible types of intervals.

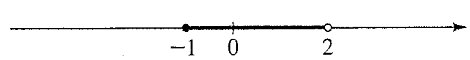
Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \le x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

The symbol ∞ ("infinity") does not stand for a number. The notation (a, ∞), for instance, simply indicates that the interval has no endpoint on the right but extends infinitely far in the positive direction.

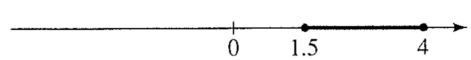
EXAMPLE 5 Graphing Intervals

Express each interval in terms of inequalities, and then graph the interval.

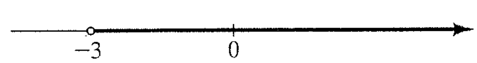
(a) $[-1, 2) = \{x \mid -1 \le x < 2\}$



(b) $[1.5, 4] = \{x \mid 1.5 \le x \le 4\}$

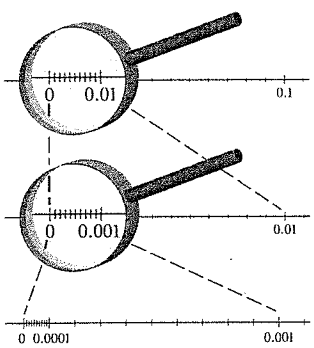


(c) $(-3, \infty) = \{x \mid -3 < x\}$



Now Try Exercise 47

No Smallest or Largest Number in an Open Interval
Any interval contains infinitely many numbers—every point on the graph of an interval corresponds to a real number. In the closed interval $[0, 1]$, the smallest number is 0 and the largest is 1, but the open interval $(0, 1)$ contains no smallest or largest number. To see this, note that 0.01 is close to zero, but 0.001 is closer, 0.0001 is closer yet, and so on. We can always find a number in the interval $(0, 1)$ closer to zero than any given number. Since 0 itself is not in the interval, the interval contains no smallest number. Similarly, 0.99 is close to 1, but 0.999 is closer, 0.9999 closer yet, and so on. Since 1 itself is not in the interval, the interval has no largest number.



EXAMPLE 6 Finding Unions and Intersections of Intervals

Graph each set.

(a) $(1, 3) \cap [2, 7]$ **(b)** $(1, 3) \cup [2, 7]$

SOLUTION

(a) The intersection of two intervals consists of the numbers that are in both intervals. Therefore

$$\begin{aligned}(1, 3) \cap [2, 7] &= \{x \mid 1 < x < 3 \text{ and } 2 \le x \le 7\} \\ &= \{x \mid 2 \le x < 3\} = [2, 3)\end{aligned}$$

This set is illustrated in Figure 8.

(b) The union of two intervals consists of the numbers that are in either one interval or the other (or both). Therefore

$$\begin{aligned}(1, 3) \cup [2, 7] &= \{x \mid 1 < x < 3 \text{ or } 2 \le x \le 7\} \\ &= \{x \mid 1 < x \le 7\} = (1, 7]\end{aligned}$$

This set is illustrated in Figure 9.

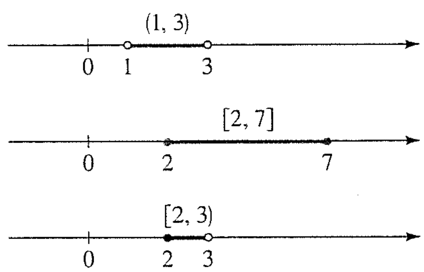


FIGURE 8 $(1, 3) \cap [2, 7] = [2, 3)$

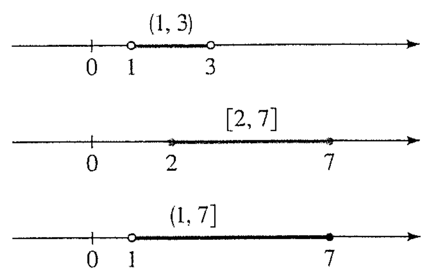


FIGURE 9 $(1, 3) \cup [2, 7] = (1, 7]$

Now Try Exercise 61

Absolute Value and Distance

The **absolute value** of a number a, denoted by $|a|$, is the distance from a to 0 on the real number line (see Figure 10). Distance is always positive or zero, so we have $|a| \ge 0$ for every number a. Remembering that $-a$ is positive when a is negative, we have the following definition.

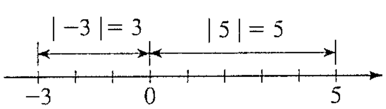


FIGURE 10

DEFINITION OF ABSOLUTE VALUE

If a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a & \text{if } a < 0 \end{cases}$$

EXAMPLE 7 Evaluating Absolute Values of Numbers

(a) $|3| = 3$

(b) $|-3| = -(-3) = 3$

(c) $|0| = 0$

(d) $|3 - \pi| = -(3 - \pi) = \pi - 3$ (since $3 < \pi \Rightarrow 3 - \pi < 0$)

Now Try Exercise 67

When working with absolute values, we use the following properties.

PROPERTIES OF ABSOLUTE VALUE

Property	Example	Description
1. $\lvert a \rvert \geq 0$	$\lvert -3 \rvert = 3 \geq 0$	The absolute value of a number is always positive or zero.
2. $\lvert a \rvert = \lvert -a \rvert$	$\lvert 5 \rvert = \lvert -5 \rvert$	A number and its negative have the same absolute value.
3. $\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	$\lvert -2 \cdot 5 \rvert = \lvert -2 \rvert \lvert 5 \rvert$	The absolute value of a product is the product of the absolute values.
4. $\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert}$	$\left\lvert \frac{12}{-3} \right\rvert = \frac{\lvert 12 \rvert}{\lvert -3 \rvert}$	The absolute value of a quotient is the quotient of the absolute values.
5. $\lvert a + b \rvert \leq \lvert a \rvert + \lvert b \rvert$	$\lvert -3 + 5 \rvert \leq \lvert -3 \rvert + \lvert 5 \rvert$	Triangle Inequality

What is the distance on the real line between the numbers -2 and 11? From Figure 11 we see that the distance is 13. We arrive at this by finding either $\lvert 11 - (-2) \rvert = 13$ or $\lvert (-2) - 11 \rvert = 13$. From this observation we make the following definition (see Figure 12).

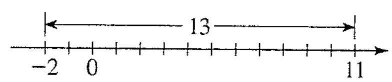


FIGURE 11

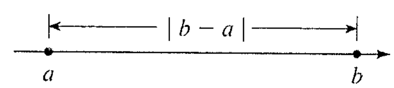


FIGURE 12 Length of a line segment is $\lvert b - a \rvert$

DISTANCE BETWEEN POINTS ON THE REAL LINE

If a and b are real numbers, then the **distance** between the points a and b on the real line is

$$d(a, b) = \lvert b - a \rvert$$

From Property 6 of negatives it follows that

$$\lvert b - a \rvert = \lvert a - b \rvert$$

This confirms that, as we would expect, the distance from a to b is the same as the distance from b to a.

EXAMPLE 8 Distance Between Points on the Real Line

The distance between the numbers -8 and 2 is

$$d(a, b) = \lvert 2 - (-8) \rvert = \lvert -10 \rvert = 10$$

We can check this calculation geometrically, as shown in Figure 13.

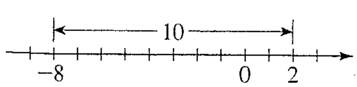


FIGURE 13

Now Try Exercise 75

1.1 EXERCISES

CONCEPTS

1. Give an example of each of the following:
   (a) A natural number
   (b) An integer that is not a natural number
   (c) A rational number that is not an integer
   (d) An irrational number

2. Complete each statement and name the property of real numbers you have used.
   (a) $ab =$ ______; ______ Property
   (b) $a + (b + c) =$ ______; ______ Property
   (c) $a(b + c) =$ ______; ______ Property

3. Express the set of real numbers between but not including 2 and 7 as follows.
   (a) In set-builder notation: ______
   (b) In interval notation: ______

4. The symbol $|x|$ stands for the ______ of the number x. If x is not 0, then the sign of $|x|$ is always ______.

5. The distance between a and b on the real line is $d(a, b) =$ ______. So the distance between -5 and 2 is ______.

6–8 ■ *Yes or No*? If *No*, give a reason. Assume that a and b are nonzero real numbers.

6. (a) Is the sum of two rational numbers always a rational number?
   (b) Is the sum of two irrational numbers always an irrational number?

7. (a) Is $a - b$ equal to $b - a$?
   (b) Is $-2(a - 5)$ equal to $-2a - 10$?

8. (a) Is the distance between any two different real numbers always positive?
   (b) Is the distance between a and b the same as the distance between b and a?

SKILLS

9–10 ■ Real Numbers List the elements of the given set that are
   (a) natural numbers
   (b) integers
   (c) rational numbers
   (d) irrational numbers

9. $\{-1.5, 0, \frac{5}{2}, \sqrt{7}, 2.71, -\pi, 3.1\overline{4}, 100, -8\}$

10. $\{1.3, 1.3333\ldots, \sqrt{5}, 5.34, -500, 1\frac{2}{3}, \sqrt{16}, \frac{246}{579}, -\frac{20}{5}\}$

11–18 ■ Properties of Real Numbers State the property of real numbers being used.

11. $3 + 7 = 7 + 3$

12. $4(2 + 3) = (2 + 3)4$

13. $(x + 2y) + 3z = x + (2y + 3z)$

14. $2(A + B) = 2A + 2B$

15. $(5x + 1)3 = 15x + 3$

16. $(x + a)(x + b) = (x + a)x + (x + a)b$

17. $2x(3 + y) = (3 + y)2x$

18. $7(a + b + c) = 7(a + b) + 7c$

19–22 ■ Properties of Real Numbers Rewrite the expression using the given property of real numbers.

19. Commutative Property of Addition, $x + 3 =$

20. Associative Property of Multiplication, $7(3x) =$

21. Distributive Property, $4(A + B) =$

22. Distributive Property, $5x + 5y =$

23–28 ■ Properties of Real Numbers Use properties of real numbers to write the expression without parentheses.

23. $3(x + y)$
24. $(a - b)8$
25. $4(2m)$
26. $\frac{4}{3}(-6y)$
27. $-\frac{5}{2}(2x - 4y)$
28. $(3a)(b + c - 2d)$

29–32 ■ Arithmetic Operations Perform the indicated operations.

29. (a) $\frac{3}{10} + \frac{4}{15}$ (b) $\frac{1}{4} + \frac{1}{5}$

30. (a) $\frac{2}{3} - \frac{3}{5}$ (b) $1 + \frac{5}{8} - \frac{1}{6}$

31. (a) $\frac{2}{3}(6 - \frac{3}{2})$ (b) $(3 + \frac{1}{4})(1 - \frac{4}{5})$

32. (a) $\dfrac{2}{\frac{2}{3}} - \dfrac{\frac{2}{3}}{2}$ (b) $\dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}}$

33–34 ■ Inequalities Place the correct symbol ($<$, $>$, or $=$) in the space.

33. (a) $3 \quad \frac{7}{2}$ (b) $-3 \quad -\frac{7}{2}$ (c) $3.5 \quad \frac{7}{2}$

34. (a) $\frac{2}{3} \quad 0.67$ (b) $\frac{2}{3} \quad -0.67$
    (c) $|0.67| \quad |-0.67|$

35–38 ■ Inequalities State whether each inequality is true or false.

35. (a) $-3 < -4$ (b) $3 < 4$

36. (a) $\sqrt{3} > 1.7325$ (b) $1.732 \geq \sqrt{3}$

37. (a) $\frac{10}{2} \geq 5$ (b) $\frac{6}{10} \geq \frac{5}{6}$

38. (a) $\frac{7}{11} \geq \frac{8}{13}$ (b) $-\frac{3}{5} > -\frac{3}{4}$

39–40 ■ Inequalities Write each statement in terms of inequalities.

39. (a) x is positive.
    (b) t is less than 4.
    (c) a is greater than or equal to π.
    (d) x is less than $\frac{1}{3}$ and is greater than -5.
    (e) The distance from p to 3 is at most 5.

40. **(a)** y is negative.
(b) z is greater than 1.
(c) b is at most 8.
(d) w is positive and is less than or equal to 17.
(e) y is at least 2 units from π.

41–44 ■ Sets Find the indicated set if

$$A = \{1, 2, 3, 4, 5, 6, 7\} \qquad B = \{2, 4, 6, 8\}$$
$$C = \{7, 8, 9, 10\}$$

41. **(a)** $A \cup B$ **(b)** $A \cap B$

42. **(a)** $B \cup C$ **(b)** $B \cap C$

43. **(a)** $A \cup C$ **(b)** $A \cap C$

44. **(a)** $A \cup B \cup C$ **(b)** $A \cap B \cap C$

45–46 ■ Sets Find the indicated set if

$$A = \{x \mid x \geq -2\} \qquad B = \{x \mid x < 4\}$$
$$C = \{x \mid -1 < x \leq 5\}$$

45. **(a)** $B \cup C$ **(b)** $B \cap C$

46. **(a)** $A \cap C$ **(b)** $A \cap B$

47–52 ■ Intervals Express the interval in terms of inequalities, and then graph the interval.

47. $(-3, 0)$ **48.** $(2, 8]$

49. $[2, 8)$ **50.** $\left[-6, -\frac{1}{2}\right]$

51. $[2, \infty)$ **52.** $(-\infty, 1)$

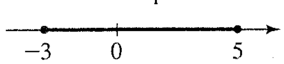
53–58 ■ Intervals Express the inequality in interval notation, and then graph the corresponding interval.

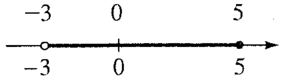
53. $x \leq 1$ **54.** $1 \leq x \leq 2$

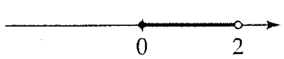
55. $-2 < x \leq 1$ **56.** $x \geq -5$

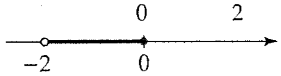
57. $x > -1$ **58.** $-5 < x < 2$

59–60 ■ Intervals Express each set in interval notation.

59. **(a)**


(b)


60. **(a)**


(b)


61–66 ■ Intervals Graph the set.

61. $(-2, 0) \cup (-1, 1)$ **62.** $(-2, 0) \cap (-1, 1)$

63. $[-4, 6] \cap [0, 8)$ **64.** $[-4, 6) \cup [0, 8)$

65. $(-\infty, -4) \cup (4, \infty)$ **66.** $(-\infty, 6] \cap (2, 10)$

67–72 ■ Absolute Value Evaluate each expression.

67. **(a)** $|100|$ **(b)** $|-73|$

68. **(a)** $|\sqrt{5} - 5|$ **(b)** $|10 - \pi|$

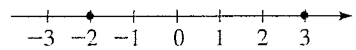
69. **(a)** $\big||-6| - |-4|\big|$ **(b)** $\dfrac{-1}{|-1|}$

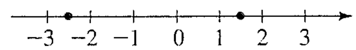
70. **(a)** $\big|2 - |-12|\big|$ **(b)** $-1 - \big|1 - |-1|\big|$

71. **(a)** $|(-2) \cdot 6|$ **(b)** $\left|\left(-\frac{1}{3}\right)(-15)\right|$

72. **(a)** $\left|\dfrac{-6}{24}\right|$ **(b)** $\left|\dfrac{7 - 12}{12 - 7}\right|$

73–76 ■ Distance Find the distance between the given numbers.

73.


74.


75. **(a)** 2 and 17 **(b)** -3 and 21 **(c)** $\frac{11}{8}$ and $-\frac{3}{10}$

76. **(a)** $\frac{7}{15}$ and $-\frac{1}{21}$ **(b)** -38 and -57 **(c)** -2.6 and -1.8

SKILLS Plus

77–78 ■ Repeating Decimal Express each repeating decimal as a fraction. (See the margin note on page 3.)

77. **(a)** $0.\overline{7}$ **(b)** $0.2\overline{8}$ **(c)** $0.\overline{57}$

78. **(a)** $5.\overline{23}$ **(b)** $1.3\overline{7}$ **(c)** $2.\overline{135}$

79–82 ■ Simplifying Absolute Value Express the quantity without using absolute value.

79. $|\pi - 3|$ **80.** $|1 - \sqrt{2}|$

81. $|a - b|$, where $a < b$

82. $a + b + |a - b|$, where $a < b$

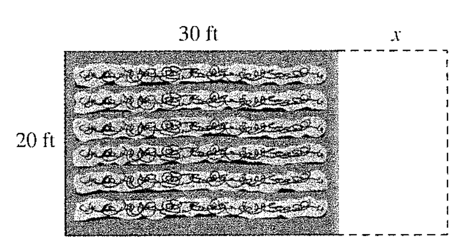
83–84 ■ Signs of Numbers Let a, b, and c be real numbers such that $a > 0$, $b < 0$, and $c < 0$. Find the sign of each expression.

83. **(a)** $-a$ **(b)** bc **(c)** $a - b$ **(d)** $ab + ac$

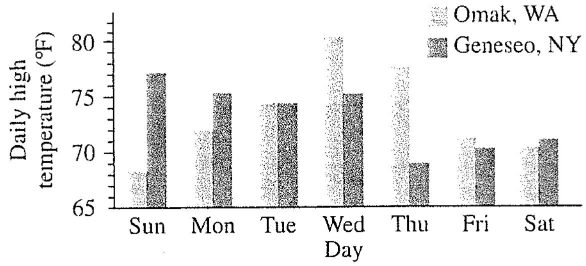
84. **(a)** $-b$ **(b)** $a + bc$ **(c)** $c - a$ **(d)** ab^2

APPLICATIONS

85. Area of a Garden Mary's backyard vegetable garden measures 20 ft by 30 ft, so its area is $20 \times 30 = 600\ \text{ft}^2$. She decides to make it longer, as shown in the figure, so that the area increases to $A = 20(30 + x)$. Which property of real numbers tells us that the new area can also be written $A = 600 + 20x$?



86. Temperature Variation The bar graph shows the daily high temperatures for Omak, Washington, and Geneseo, New York, during a certain week in June. Let T_O represent the temperature in Omak and T_G the temperature in Geneseo. Calculate $T_O - T_G$ and $|T_O - T_G|$ for each day shown. Which of these two values gives more information?

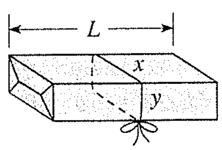


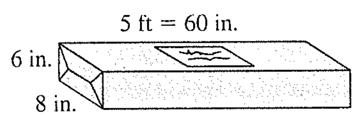
87. Mailing a Package The post office will accept only packages for which the length plus the "girth" (distance around) is no more than 108 in. Thus for the package in the figure, we must have

$$L + 2(x + y) \le 108$$

(a) Will the post office accept a package that is 6 in. wide, 8 in. deep, and 5 ft long? What about a package that measures 2 ft by 2 ft by 4 ft?

(b) What is the greatest acceptable length for a package that has a square base measuring 9 in. by 9 in.?





DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

88. DISCUSS: Sums and Products of Rational and Irrational Numbers Explain why the sum, the difference, and the product of two rational numbers are rational numbers. Is the product of two irrational numbers necessarily irrational? What about the sum?

89. DISCOVER ■ PROVE: Combining Rational and Irrational Numbers Is $\frac{1}{2} + \sqrt{2}$ rational or irrational? Is $\frac{1}{2} \cdot \sqrt{2}$ rational or irrational? Experiment with sums and products of other rational and irrational numbers. Prove the following.

(a) The sum of a rational number r and an irrational number t is irrational.

(b) The product of a rational number r and an irrational number t is irrational.

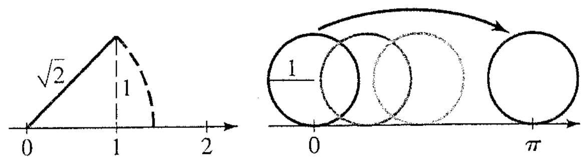
[*Hint:* For part (a), suppose that $r + t$ is a rational number q, that is, $r + t = q$. Show that this leads to a contradiction. Use similar reasoning for part (b).]

90. DISCOVER: Limiting Behavior of Reciprocals Complete the tables. What happens to the size of the fraction $1/x$ as x gets large? As x gets small?

x	$1/x$
1	
2	
10	
100	
1000	

x	$1/x$
1.0	
0.5	
0.1	
0.01	
0.001	

91. DISCOVER: Locating Irrational Numbers on the Real Line Using the figures below, explain how to locate the point $\sqrt{2}$ on a number line. Can you locate $\sqrt{5}$ by a similar method? How can the circle shown in the figure help us to locate π on a number line? List some other irrational numbers that you can locate on a number line.



92. PROVE: Maximum and Minimum Formulas Let $\max(a, b)$ denote the maximum and $\min(a, b)$ denote the minimum of the real numbers a and b. For example, $\max(2, 5) = 5$ and $\min(-1, -2) = -2$.

(a) Prove that $\max(a, b) = \dfrac{a + b + |a - b|}{2}$.

(b) Prove that $\min(a, b) = \dfrac{a + b - |a - b|}{2}$.

[*Hint:* Take cases and write these expressions without absolute values. See Exercises 81 and 82.]

93. WRITE: Real Numbers in the Real World Write a paragraph describing different real-world situations in which you would use natural numbers, integers, rational numbers, and irrational numbers. Give examples for each type of situation.

94. DISCUSS: Commutative and Noncommutative Operations We have learned that addition and multiplication are both commutative operations.

(a) Is subtraction commutative?

(b) Is division of nonzero real numbers commutative?

(c) Are the actions of putting on your socks and putting on your shoes commutative?

(d) Are the actions of putting on your hat and putting on your coat commutative?

(e) Are the actions of washing laundry and drying it commutative?

95. PROVE: Triangle Inequality We prove Property 5 of absolute values, the Triangle Inequality:

$$|x + y| \le |x| + |y|$$

(a) Verify that the Triangle Inequality holds for $x = 2$ and $y = 3$, for $x = -2$ and $y = -3$, and for $x = -2$ and $y = 3$.

(b) Prove that the Triangle Inequality is true for all real numbers x and y. [*Hint:* Take cases.]

1.2 EXPONENTS AND RADICALS

Integer Exponents ▪ Rules for Working with Exponents ▪ Scientific Notation ▪ Radicals ▪ Rational Exponents ▪ Rationalizing the Denominator; Standard Form

In this section we give meaning to expressions such as $a^{m/n}$ in which the exponent m/n is a rational number. To do this, we need to recall some facts about integer exponents, radicals, and nth roots.

Integer Exponents

A product of identical numbers is usually written in exponential notation. For example, $5 \cdot 5 \cdot 5$ is written as 5^3. In general, we have the following definition.

EXPONENTIAL NOTATION

If a is any real number and n is a positive integer, then the ***n*th power** of a is

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$$

The number a is called the **base**, and n is called the **exponent**.

EXAMPLE 1 Exponential Notation

(a) $\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{32}$

(b) $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$

(c) $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

Now Try Exercise 17

Note the distinction between $(-3)^4$ and -3^4. In $(-3)^4$ the exponent applies to -3, but in -3^4 the exponent applies only to 3.

We can state several useful rules for working with exponential notation. To discover the rule for multiplication, we multiply 5^4 by 5^2:

$$5^4 \cdot 5^2 = (\underbrace{5 \cdot 5 \cdot 5 \cdot 5}_{4 \text{ factors}})(\underbrace{5 \cdot 5}_{2 \text{ factors}}) = \underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}_{6 \text{ factors}} = 5^6 = 5^{4+2}$$

It appears that *to multiply two powers of the same base, we add their exponents.* In general, for any real number a and any positive integers m and n, we have

$$a^m a^n = (\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}})(\underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}) = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{m+n \text{ factors}} = a^{m+n}$$

Thus $a^m a^n = a^{m+n}$.

We would like this rule to be true even when m and n are 0 or negative integers. For instance, we must have

$$2^0 \cdot 2^3 = 2^{0+3} = 2^3$$

But this can happen only if $2^0 = 1$. Likewise, we want to have

$$5^4 \cdot 5^{-4} = 5^{4+(-4)} = 5^{4-4} = 5^0 = 1$$

and this will be true if $5^{-4} = 1/5^4$. These observations lead to the following definition.

ZERO AND NEGATIVE EXPONENTS

If $a \neq 0$ is a real number and n is a positive integer, then

$$a^0 = 1 \qquad \text{and} \qquad a^{-n} = \frac{1}{a^n}$$

EXAMPLE 2 Zero and Negative Exponents

(a) $\left(\frac{4}{7}\right)^0 = 1$

(b) $x^{-1} = \dfrac{1}{x^1} = \dfrac{1}{x}$

(c) $(-2)^{-3} = \dfrac{1}{(-2)^3} = \dfrac{1}{-8} = -\dfrac{1}{8}$

Now Try Exercise 19

Rules for Working with Exponents

Familiarity with the following rules is essential for our work with exponents and bases. In the table the bases a and b are real numbers, and the exponents m and n are integers.

LAWS OF EXPONENTS

Law	Example	Description
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^5 = 3^{2+5} = 3^7$	To multiply two powers of the same number, add the exponents.
2. $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{3^5}{3^2} = 3^{5-2} = 3^3$	To divide two powers of the same number, subtract the exponents.
3. $(a^m)^n = a^{mn}$	$(3^2)^5 = 3^{2 \cdot 5} = 3^{10}$	To raise a power to a new power, multiply the exponents.
4. $(ab)^n = a^n b^n$	$(3 \cdot 4)^2 = 3^2 \cdot 4^2$	To raise a product to a power, raise each factor to the power.
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{3}{4}\right)^2 = \dfrac{3^2}{4^2}$	To raise a quotient to a power, raise both numerator and denominator to the power.
6. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$	$\left(\dfrac{3}{4}\right)^{-2} = \left(\dfrac{4}{3}\right)^2$	To raise a fraction to a negative power, invert the fraction and change the sign of the exponent.
7. $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$	$\dfrac{3^{-2}}{4^{-5}} = \dfrac{4^5}{3^2}$	To move a number raised to a power from numerator to denominator or from denominator to numerator, change the sign of the exponent.

Proof of Law 3 If m and n are positive integers, we have

$$(a^m)^n = (\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}})^n$$

$$= \underbrace{(\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}})(\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}}) \cdots (\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}})}_{n \text{ groups of factors}}$$

$$= \underbrace{a \cdot a \cdot \cdots \cdot a}_{mn \text{ factors}} = a^{mn}$$

The cases for which $m \le 0$ or $n \le 0$ can be proved by using the definition of negative exponents.

Proof of Law 4 If n is a positive integer, we have

$$(ab)^n = \underbrace{(ab)(ab)\cdots(ab)}_{n\text{ factors}} = \underbrace{(a\cdot a\cdot\cdots\cdot a)}_{n\text{ factors}}\cdot\underbrace{(b\cdot b\cdot\cdots\cdot b)}_{n\text{ factors}} = a^n b^n$$

Here we have used the Commutative and Associative Properties repeatedly. If $n \le 0$, Law 4 can be proved by using the definition of negative exponents. ■

You are asked to prove Laws 2, 5, 6, and 7 in Exercises 108 and 109.

EXAMPLE 3 Using Laws of Exponents

(a) $x^4x^7 = x^{4+7} = x^{11}$ Law 1: $a^m a^n = a^{m+n}$

(b) $y^4y^{-7} = y^{4-7} = y^{-3} = \dfrac{1}{y^3}$ Law 1: $a^m a^n = a^{m+n}$

(c) $\dfrac{c^9}{c^5} = c^{9-5} = c^4$ Law 2: $\dfrac{a^m}{a^n} = a^{m-n}$

(d) $(b^4)^5 = b^{4\cdot 5} = b^{20}$ Law 3: $(a^m)^n = a^{mn}$

(e) $(3x)^3 = 3^3x^3 = 27x^3$ Law 4: $(ab)^n = a^nb^n$

(f) $\left(\dfrac{x}{2}\right)^5 = \dfrac{x^5}{2^5} = \dfrac{x^5}{32}$ Law 5: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Now Try Exercises 29, 31, and 33 ■

EXAMPLE 4 Simplifying Expressions with Exponents

Simplify:

(a) $(2a^3b^2)(3ab^4)^3$ **(b)** $\left(\dfrac{x}{y}\right)^3\left(\dfrac{y^2x}{z}\right)^4$

SOLUTION

(a)
$$\begin{aligned}(2a^3b^2)(3ab^4)^3 &= (2a^3b^2)[3^3a^3(b^4)^3] && \text{Law 4: } (ab)^n = a^nb^n\\ &= (2a^3b^2)(27a^3b^{12}) && \text{Law 3: } (a^m)^n = a^{mn}\\ &= (2)(27)a^3a^3b^2b^{12} && \text{Group factors with the same base}\\ &= 54a^6b^{14} && \text{Law 1: } a^ma^n = a^{m+n}\end{aligned}$$

(b)
$$\begin{aligned}\left(\frac{x}{y}\right)^3\left(\frac{y^2x}{z}\right)^4 &= \frac{x^3}{y^3}\,\frac{(y^2)^4x^4}{z^4} && \text{Laws 5 and 4}\\ &= \frac{x^3}{y^3}\,\frac{y^8x^4}{z^4} && \text{Law 3}\\ &= (x^3x^4)\left(\frac{y^8}{y^3}\right)\frac{1}{z^4} && \text{Group factors with the same base}\\ &= \frac{x^7y^5}{z^4} && \text{Laws 1 and 2}\end{aligned}$$

Now Try Exercises 35 and 39 ■

When simplifying an expression, you will find that many different methods will lead to the same result; you should feel free to use any of the rules of exponents to arrive at your own method. In the next example we see how to simplify expressions with negative exponents.

Mathematics in the Modern World

Although we are often unaware of its presence, mathematics permeates nearly every aspect of life in the modern world. With the advent of modern technology, mathematics plays an ever greater role in our lives. Today you were probably awakened by a digital alarm clock, sent a text, surfed the Internet, watched HDTV or a streaming video, listened to music on your cell phone, drove a car with digitally controlled fuel injection, then fell asleep in a room whose temperature is controlled by a digital thermostat. In each of these activities mathematics is crucially involved. In general, a property such as the intensity or frequency of sound, the oxygen level in the exhaust emission from a car, the colors in an image, or the temperature in your bedroom is transformed into sequences of numbers by sophisticated mathematical algorithms. These numerical data, which usually consist of many millions of bits (the digits 0 and 1), are then transmitted and reinterpreted. Dealing with such huge amounts of data was not feasible until the invention of computers, machines whose logical processes were invented by mathematicians.

The contributions of mathematics in the modern world are not limited to technological advances. The logical processes of mathematics are now used to analyze complex problems in the social, political, and life sciences in new and surprising ways. Advances in mathematics continue to be made, some of the most exciting of these just within the past decade.

In other *Mathematics in the Modern World,* we will describe in more detail how mathematics affects all of us in our everyday activities.

EXAMPLE 5 Simplifying Expressions with Negative Exponents

Eliminate negative exponents, and simplify each expression.

(a) $\dfrac{6st^{-4}}{2s^{-2}t^2}$ **(b)** $\left(\dfrac{y}{3z^3}\right)^{-2}$

SOLUTION

(a) We use Law 7, which allows us to move a number raised to a power from the numerator to the denominator (or vice versa) by changing the sign of the exponent.

t^{-4} moves to denominator and becomes t^4.

s^{-2} moves to numerator and becomes s^2

$$\frac{6st^{-4}}{2s^{-2}t^2} = \frac{6ss^2}{2t^2t^4} \qquad \text{Law 7}$$

$$= \frac{3s^3}{t^6} \qquad \text{Law 1}$$

(b) We use Law 6, which allows us to change the sign of the exponent of a fraction by inverting the fraction.

$$\left(\frac{y}{3z^3}\right)^{-2} = \left(\frac{3z^3}{y}\right)^2 \qquad \text{Law 6}$$

$$= \frac{9z^6}{y^2} \qquad \text{Laws 5 and 4}$$

Now Try Exercise 41 ■

Scientific Notation

Scientists use exponential notation as a compact way of writing very large numbers and very small numbers. For example, the nearest star beyond the sun, Proxima Centauri, is approximately 40,000,000,000,000 km away. The mass of a hydrogen atom is about 0.00000000000000000000000166 g. Such numbers are difficult to read and to write, so scientists usually express them in *scientific notation.*

SCIENTIFIC NOTATION

A positive number x is said to be written in **scientific notation** if it is expressed as follows:

$$x = a \times 10^n \qquad \text{where } 1 \le a < 10 \text{ and } n \text{ is an integer}$$

For instance, when we state that the distance to the star Proxima Centauri is 4×10^{13} km, the positive exponent 13 indicates that the decimal point should be moved 13 places to the *right*:

$$4 \times 10^{13} = 40{,}000{,}000{,}000{,}000$$

Move decimal point 13 places to the right

When we state that the mass of a hydrogen atom is 1.66×10^{-24} g, the exponent -24 indicates that the decimal point should be moved 24 places to the *left*:

$$1.66 \times 10^{-24} = 0.00000000000000000000000166$$

Move decimal point 24 places to the left

EXAMPLE 6 Changing from Decimal to Scientific Notation

Write each number in scientific notation.

(a) 56,920 **(b)** 0.000093

SOLUTION

(a) $\underbrace{56{,}920}_{\text{4 places}} = 5.692 \times 10^4$ **(b)** $\underbrace{0.000093}_{\text{5 places}} = 9.3 \times 10^{-5}$

Now Try Exercise 83

EXAMPLE 7 Changing from Scientific Notation to Decimal Notation

Write each number in decimal notation.

(a) 6.97×10^9 **(b)** 4.6271×10^{-6}

SOLUTION

(a) $6.97 \times 10^9 = \underbrace{6{,}970{,}000{,}000}_{\text{9 places}}$ Move decimal 9 places to the right

(b) $4.6271 \times 10^{-6} = \underbrace{0.0000046271}_{\text{6 places}}$ Move decimal 6 places to the left

Now Try Exercise 85

To use scientific notation on a calculator, press the key labeled EE or EXP or EEX to enter the exponent. For example, to enter the number 3.629×10^{15} on a TI-83 or TI-84 calculator, we enter

3.629 2ND EE 15

and the display reads

3.629E15

Scientific notation is often used on a calculator to display a very large or very small number. For instance, if we use a calculator to square the number 1,111,111, the display panel may show (depending on the calculator model) the approximation

1.234568 12 or 1.234568 E12

Here the final digits indicate the power of 10, and we interpret the result as

$$1.234568 \times 10^{12}$$

EXAMPLE 8 Calculating with Scientific Notation

If $a \approx 0.00046$, $b \approx 1.697 \times 10^{22}$, and $c \approx 2.91 \times 10^{-18}$, use a calculator to approximate the quotient ab/c.

SOLUTION We could enter the data using scientific notation, or we could use laws of exponents as follows:

$$\begin{aligned} \frac{ab}{c} &\approx \frac{(4.6 \times 10^{-4})(1.697 \times 10^{22})}{2.91 \times 10^{-18}} \\ &= \frac{(4.6)(1.697)}{2.91} \times 10^{-4+22+18} \\ &\approx 2.7 \times 10^{36} \end{aligned}$$

We state the answer rounded to two significant figures because the least accurate of the given numbers is stated to two significant figures.

For guidelines on working with significant figures, see Appendix B, *Calculations and Significant Figures*.

Now Try Exercises 89 and 91

Radicals

We know what 2^n means whenever n is an integer. To give meaning to a power, such as $2^{4/5}$, whose exponent is a rational number, we need to discuss radicals.

The symbol $\sqrt{\ }$ means "the positive square root of." Thus

$$\sqrt{a} = b \quad \text{means} \quad b^2 = a \quad \text{and} \quad b \ge 0$$

It is true that the number 9 has two square roots, 3 and -3, but the notation $\sqrt{9}$ is reserved for the *positive* square root of 9 (sometimes called the *principal square root* of 9). If we want the negative root, we must write $-\sqrt{9}$, which is -3.

Since $a = b^2 \ge 0$, the symbol $\sqrt{a}$ makes sense only when $a \ge 0$. For instance,

$$\sqrt{9} = 3 \quad \text{because} \quad 3^2 = 9 \quad \text{and} \quad 3 \ge 0$$

Square roots are special cases of nth roots. The nth root of x is the number that, when raised to the nth power, gives x.

DEFINITION OF nth ROOT

If n is any positive integer, then the **principal nth root** of a is defined as follows:

$$\sqrt[n]{a} = b \quad \text{means} \quad b^n = a$$

If n is even, we must have $a \ge 0$ and $b \ge 0$.

For example,

$$\sqrt[4]{81} = 3 \quad \text{because} \quad 3^4 = 81 \quad \text{and} \quad 3 \ge 0$$

$$\sqrt[3]{-8} = -2 \quad \text{because} \quad (-2)^3 = -8$$

But $\sqrt{-8}$, $\sqrt[4]{-8}$, and $\sqrt[6]{-8}$ are not defined. (For instance, $\sqrt{-8}$ is not defined because the square of every real number is nonnegative.)

Notice that

$$\sqrt{4^2} = \sqrt{16} = 4 \quad \text{but} \quad \sqrt{(-4)^2} = \sqrt{16} = 4 = |-4|$$

So the equation $\sqrt{a^2} = a$ is not always true; it is true only when $a \ge 0$. However, we can always write $\sqrt{a^2} = |a|$. This last equation is true not only for square roots, but for any even root. This and other rules used in working with nth roots are listed in the following box. In each property we assume that all the given roots exist.

PROPERTIES OF nth ROOTS

Property	Example
1. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$	$\sqrt[3]{-8 \cdot 27} = \sqrt[3]{-8}\sqrt[3]{27} = (-2)(3) = -6$
2. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt[4]{\dfrac{16}{81}} = \dfrac{\sqrt[4]{16}}{\sqrt[4]{81}} = \dfrac{2}{3}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt{\sqrt[3]{729}} = \sqrt[6]{729} = 3$
4. $\sqrt[n]{a^n} = a$ if n is odd	$\sqrt[3]{(-5)^3} = -5$, $\sqrt[5]{2^5} = 2$
5. $\sqrt[n]{a^n} = \lvert a \rvert$ if n is even	$\sqrt[4]{(-3)^4} = \lvert -3 \rvert = 3$

EXAMPLE 9 Simplifying Expressions Involving nth Roots

(a) $\sqrt[3]{x^4} = \sqrt[3]{x^3 x}$ Factor out the largest cube

$= \sqrt[3]{x^3}\sqrt[3]{x}$ Property 1: $\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$

$= x\sqrt[3]{x}$ Property 4: $\sqrt[3]{a^3} = a$

(b) $\sqrt[4]{81x^8y^4} = \sqrt[4]{81}\sqrt[4]{x^8}\sqrt[4]{y^4}$ Property 1: $\sqrt[4]{abc} = \sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{c}$

$= 3\sqrt[4]{(x^2)^4}|y|$ Property 5: $\sqrt[4]{a^4} = |a|$

$= 3x^2|y|$ Property 5: $\sqrt[4]{a^4} = |a|$, $|x^2| = x^2$

Now Try Exercises 45 and 47

It is frequently useful to combine like radicals in an expression such as $2\sqrt{3} + 5\sqrt{3}$. This can be done by using the Distributive Property. For example,

$$2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3} = 7\sqrt{3}$$

The next example further illustrates this process.

Avoid making the following error:

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

For instance, if we let $a = 9$ and $b = 16$, then we see the error:

$\sqrt{9+16} \stackrel{?}{=} \sqrt{9} + \sqrt{16}$

$\sqrt{25} \stackrel{?}{=} 3 + 4$

$5 \stackrel{?}{=} 7$ Wrong!

EXAMPLE 10 Combining Radicals

(a) $\sqrt{32} + \sqrt{200} = \sqrt{16 \cdot 2} + \sqrt{100 \cdot 2}$ Factor out the largest squares

$= \sqrt{16}\sqrt{2} + \sqrt{100}\sqrt{2}$ Property 1: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 4\sqrt{2} + 10\sqrt{2} = 14\sqrt{2}$ Distributive Property

(b) If $b > 0$, then

$\sqrt{25b} - \sqrt{b^3} = \sqrt{25}\sqrt{b} - \sqrt{b^2}\sqrt{b}$ Property 1: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 5\sqrt{b} - b\sqrt{b}$ Property 5, $b > 0$

$= (5 - b)\sqrt{b}$ Distributive Property

(c) $\sqrt{49x^2 + 49} = \sqrt{49(x^2 + 1)}$ Factor out the perfect square

$= 7\sqrt{x^2 + 1}$ Property 1: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Now Try Exercises 49, 51, and 53

Rational Exponents

To define what is meant by a *rational exponent* or, equivalently, a *fractional exponent* such as $a^{1/3}$, we need to use radicals. To give meaning to the symbol $a^{1/n}$ in a way that is consistent with the Laws of Exponents, we would have to have

$$(a^{1/n})^n = a^{(1/n)n} = a^1 = a$$

So by the definition of nth root,

$$a^{1/n} = \sqrt[n]{a}$$

In general, we define rational exponents as follows.

DEFINITION OF RATIONAL EXPONENTS

For any rational exponent m/n in lowest terms, where m and n are integers and $n > 0$, we define

$$a^{m/n} = (\sqrt[n]{a})^m \quad \text{or equivalently} \quad a^{m/n} = \sqrt[n]{a^m}$$

If n is even, then we require that $a \geq 0$.

With this definition it can be proved that *the Laws of Exponents also hold for rational exponents.*

DIOPHANTUS lived in Alexandria about 250 A.D. His book *Arithmetica* is considered the first book on algebra. In it he gives methods for finding integer solutions of algebraic equations. *Arithmetica* was read and studied for more than a thousand years. Fermat (see page 117) made some of his most important discoveries while studying this book. Diophantus' major contribution is the use of symbols to stand for the unknowns in a problem. Although his symbolism is not as simple as what we use today, it was a major advance over writing everything in words. In Diophantus' notation the equation

$$x^5 - 7x^2 + 8x - 5 = 24$$

is written

ΔΚ^γ α ς η ⫛ Δ^γ ζ M̊ ε ι^σ κ δ

Our modern algebraic notation did not come into common use until the 17th century.

EXAMPLE 11 Using the Definition of Rational Exponents

(a) $4^{1/2} = \sqrt{4} = 2$

(b) $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$ Alternative solution: $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

(c) $125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \dfrac{1}{5}$

Now Try Exercises 55 and 57

EXAMPLE 12 Using the Laws of Exponents with Rational Exponents

(a) $a^{1/3}a^{7/3} = a^{8/3}$ — Law 1: $a^m a^n = a^{m+n}$

(b) $\dfrac{a^{2/5}a^{7/5}}{a^{3/5}} = a^{2/5+7/5-3/5} = a^{6/5}$ — Law 1, Law 2: $\dfrac{a^m}{a^n} = a^{m-n}$

(c) $(2a^3b^4)^{3/2} = 2^{3/2}(a^3)^{3/2}(b^4)^{3/2}$ — Law 4: $(abc)^n = a^n b^n c^n$

$= (\sqrt{2})^3 a^{3(3/2)} b^{4(3/2)}$ — Law 3: $(a^m)^n = a^{mn}$

$= 2\sqrt{2}a^{9/2}b^6$

(d) $\left(\dfrac{2x^{3/4}}{y^{1/3}}\right)^3\left(\dfrac{y^4}{x^{-1/2}}\right) = \dfrac{2^3(x^{3/4})^3}{(y^{1/3})^3}\cdot(y^4x^{1/2})$ — Laws 5, 4, and 7

$= \dfrac{8x^{9/4}}{y}\cdot y^4x^{1/2}$ — Law 3

$= 8x^{11/4}y^3$ — Laws 1 and 2

Now Try Exercises 61, 63, 67, and 69

EXAMPLE 13 Simplifying by Writing Radicals as Rational Exponents

(a) $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$ — Definition of rational and negative exponents

(b) $(2\sqrt{x})(3\sqrt[3]{x}) = (2x^{1/2})(3x^{1/3})$ — Definition of rational exponents

$= 6x^{1/2+1/3} = 6x^{5/6}$ — Law 1

(c) $\sqrt{x\sqrt{x}} = (xx^{1/2})^{1/2}$ — Definition of rational exponents

$= (x^{3/2})^{1/2}$ — Law 1

$= x^{3/4}$ — Law 3

Now Try Exercises 73 and 77

Rationalizing the Denominator; Standard Form

It is often useful to eliminate the radical in a denominator by multiplying both numerator and denominator by an appropriate expression. This procedure is called **rationalizing the denominator**. If the denominator is of the form $\sqrt{a}$, we multiply numerator and denominator by $\sqrt{a}$. In doing this we multiply the given quantity by 1, so we do not change its value. For instance,

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}\cdot 1 = \frac{1}{\sqrt{a}}\cdot\frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

Note that the denominator in the last fraction contains no radical. In general, if the denominator is of the form $\sqrt[n]{a^m}$ with $m < n$, then multiplying the numerator and denominator by $\sqrt[n]{a^{n-m}}$ will rationalize the denominator, because (for $a > 0$)

$$\sqrt[n]{a^m}\sqrt[n]{a^{n-m}} = \sqrt[n]{a^{m+n-m}} = \sqrt[n]{a^n} = a$$

A fractional expression whose denominator contains no radicals is said to be in **standard form**.

EXAMPLE 14 Rationalizing Denominators

Put each fractional expression into standard form by rationalizing the denominator.

(a) $\dfrac{2}{\sqrt{3}}$ **(b)** $\dfrac{1}{\sqrt[3]{5}}$ **(c)** $\sqrt[7]{\dfrac{1}{a^2}}$

SOLUTION

This equals 1

(a) $\dfrac{2}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{2\sqrt{3}}{3}$ $\sqrt{3} \cdot \sqrt{3} = 3$

(b) $\dfrac{1}{\sqrt[3]{5}} = \dfrac{1}{\sqrt[3]{5}} \cdot \dfrac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$ Multiply by $\dfrac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$

$= \dfrac{\sqrt[3]{25}}{5}$ $\sqrt[3]{5} \cdot \sqrt[3]{5^2} = \sqrt[3]{5^3} = 5$

(c) $\sqrt[7]{\dfrac{1}{a^2}} = \dfrac{1}{\sqrt[7]{a^2}}$ Property 2: $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$= \dfrac{1}{\sqrt[7]{a^2}} \cdot \dfrac{\sqrt[7]{a^5}}{\sqrt[7]{a^5}}$ Multiply by $\dfrac{\sqrt[7]{a^5}}{\sqrt[7]{a^5}}$

$= \dfrac{\sqrt[7]{a^5}}{a}$ $\sqrt[7]{a^2} \cdot \sqrt[7]{a^5} = a$

Now Try Exercises 79 and 81

1.2 EXERCISES

CONCEPTS

1. (a) Using exponential notation, we can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as ________.
   (b) In the expression 3^4 the number 3 is called the ________, and the number 4 is called the ________.
2. (a) When we multiply two powers with the same base, we ________ the exponents. So $3^4 \cdot 3^5 =$ ________.
   (b) When we divide two powers with the same base, we ________ the exponents. So $\dfrac{3^5}{3^2} =$ ________.
3. (a) Using exponential notation, we can write $\sqrt[3]{5}$ as ________.
   (b) Using radicals, we can write $5^{1/2}$ as ________.
   (c) Is there a difference between $\sqrt{5^2}$ and $(\sqrt{5})^2$? Explain.
4. Explain what $4^{3/2}$ means, then calculate $4^{3/2}$ in two different ways:

   $(4^{1/2})\ \ = $ ________ or $(4^3)\ \ =$ ________
5. Explain how we rationalize a denominator, then complete the following steps to rationalize $\dfrac{1}{\sqrt{3}}$:

   $$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\quad}{\quad} = \frac{\quad}{\quad}$$
6. Find the missing power in the following calculation: $5^{1/3} \cdot 5^{\quad} = 5$.

7–8 ■ *Yes or No*? If *No*, give a reason.

7. (a) Is the expression $\left(\frac{2}{3}\right)^{-2}$ equal to $\frac{3}{4}$?
   (b) Is there a difference between $(-5)^4$ and -5^4?
8. (a) Is the expression $(x^2)^3$ equal to x^5?
   (b) Is the expression $(2x^4)^3$ equal to $2x^{12}$?
   (c) Is the expression $\sqrt{4a^2}$ equal to $2a$?
   (d) Is the expression $\sqrt{a^2 + 4}$ equal to $a + 2$?

SKILLS

9–16 ■ Radicals and Exponents Write each radical expression using exponents, and each exponential expression using radicals.

	Radical expression	Exponential expression
9.	$\dfrac{1}{\sqrt{3}}$	
10.	$\sqrt[3]{7^2}$	
11.		$4^{2/3}$
12.		$10^{-3/2}$
13.	$\sqrt[5]{5^3}$	
14.		$2^{-1.5}$
15.		$a^{2/5}$
16.	$\dfrac{1}{\sqrt{x^5}}$	

17–28 ■ Radicals and Exponents Evaluate each expression.

17. (a) -2^6 (b) $(-2)^6$ (c) $\left(\frac{1}{5}\right)^2 \cdot (-3)^3$

18. (a) $(-5)^3$ (b) -5^3 (c) $(-5)^2 \cdot \left(\frac{2}{5}\right)^2$

19. (a) $\left(\frac{5}{3}\right)^0 \cdot 2^{-1}$ (b) $\dfrac{2^{-3}}{3^0}$ (c) $\left(\frac{2}{3}\right)^{-2}$

20. (a) $-2^3 \cdot (-2)^0$ (b) $-2^{-3} \cdot (-2)^0$ (c) $\left(\frac{-3}{5}\right)^{-3}$

21. (a) $5^3 \cdot 5$ (b) $5^4 \cdot 5^{-2}$ (c) $(2^2)^3$

22. (a) $3^8 \cdot 3^5$ (b) $\dfrac{10^7}{10^4}$ (c) $(3^5)^4$

23. (a) $3\sqrt[3]{16}$ (b) $\dfrac{\sqrt{18}}{\sqrt{81}}$ (c) $\sqrt{\frac{27}{4}}$

24. (a) $2\sqrt[3]{81}$ (b) $\dfrac{\sqrt{18}}{\sqrt{25}}$ (c) $\sqrt{\frac{12}{49}}$

25. (a) $\sqrt{3}\sqrt{15}$ (b) $\dfrac{\sqrt{48}}{\sqrt{3}}$ (c) $\sqrt[3]{24}\sqrt[3]{18}$

26. (a) $\sqrt{10}\sqrt{32}$ (b) $\dfrac{\sqrt{54}}{\sqrt{6}}$ (c) $\sqrt[3]{15}\sqrt[3]{75}$

27. (a) $\dfrac{\sqrt{132}}{\sqrt{3}}$ (b) $\sqrt[3]{2}\sqrt[3]{32}$ (c) $\sqrt[4]{\frac{1}{4}}\sqrt[4]{\frac{1}{64}}$

28. (a) $\sqrt[5]{\frac{1}{8}}\sqrt[5]{\frac{1}{4}}$ (b) $\sqrt[6]{\frac{1}{2}}\sqrt[6]{128}$ (c) $\dfrac{\sqrt[3]{4}}{\sqrt[3]{108}}$

29–34 ■ Exponents Simplify each expression, and eliminate any negative exponents.

29. (a) $x^3 \cdot x^4$ (b) $(2y^2)^3$ (c) $y^{-2}y^7$

30. (a) $y^5 \cdot y^2$ (b) $(8x)^2$ (c) x^4x^{-3}

31. (a) $x^{-5} \cdot x^3$ (b) $w^{-2}w^{-4}w^5$ (c) $\dfrac{x^{16}}{x^{10}}$

32. (a) $y^2 \cdot y^{-5}$ (b) $z^5z^{-3}z^{-4}$ (c) $\dfrac{y^7y^0}{y^{10}}$

33. (a) $\dfrac{a^9a^{-2}}{a}$ (b) $(a^2a^4)^3$ (c) $\left(\dfrac{x}{2}\right)^3(5x^6)$

34. (a) $\dfrac{z^2z^4}{z^3z^{-1}}$ (b) $(2a^3a^2)^4$ (c) $(-3z^2)^3(2z^3)$

35–44 ■ Exponents Simplify each expression, and eliminate any negative exponents.

35. (a) $(3x^3y^2)(2y^3)$ (b) $(5w^2z^{-2})^2(z^3)$

36. (a) $(8m^{-2}n^4)(\frac{1}{2}n^{-2})$ (b) $(3a^4b^{-2})^3(a^2b^{-1})$

37. (a) $\dfrac{x^2y^{-1}}{x^{-5}}$ (b) $\left(\dfrac{a^3}{2b^2}\right)^3$

38. (a) $\dfrac{y^{-2}z^{-3}}{y^{-1}}$ (b) $\left(\dfrac{x^3y^{-2}}{x^{-3}y^2}\right)^{-2}$

39. (a) $\left(\dfrac{a^2}{b}\right)^5\left(\dfrac{a^3b^2}{c^3}\right)^3$ (b) $\dfrac{(u^{-1}v^2)^2}{(u^3v^{-2})^3}$

40. (a) $\left(\dfrac{x^4z^2}{4y^5}\right)\left(\dfrac{2x^3y^2}{z^3}\right)^2$ (b) $\dfrac{(rs^2)^3}{(r^{-3}s^2)^2}$

41. (a) $\dfrac{8a^3b^{-4}}{2a^{-5}b^5}$ (b) $\left(\dfrac{y}{5x^{-2}}\right)^{-3}$

42. (a) $\dfrac{5xy^{-2}}{x^{-1}y^{-3}}$ (b) $\left(\dfrac{2a^{-1}b}{a^2b^{-3}}\right)^{-3}$

43. (a) $\left(\dfrac{3a}{b^3}\right)^{-1}$ (b) $\left(\dfrac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1}$

44. (a) $\left(\dfrac{s^2t^{-4}}{5s^{-1}t}\right)^{-2}$ (b) $\left(\dfrac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$

45–48 ■ Radicals Simplify the expression. Assume that the letters denote any positive real numbers.

45. (a) $\sqrt[4]{x^4}$ (b) $\sqrt[4]{16x^8}$

46. (a) $\sqrt[5]{x^{10}}$ (b) $\sqrt[3]{x^3y^6}$

47. (a) $\sqrt[6]{64a^6b^7}$ (b) $\sqrt[3]{a^2b}\sqrt[3]{64a^4b}$

48. (a) $\sqrt[4]{x^4y^2z^2}$ (b) $\sqrt[3]{\sqrt{64x^6}}$

49–54 ■ Radical Expressions Simplify the expression.

49. (a) $\sqrt{32} + \sqrt{18}$ (b) $\sqrt{75} + \sqrt{48}$

50. (a) $\sqrt{125} + \sqrt{45}$ (b) $\sqrt[3]{54} - \sqrt[3]{16}$

51. (a) $\sqrt{9a^3} + \sqrt{a}$ (b) $\sqrt{16x} + \sqrt{x^5}$

52. (a) $\sqrt[3]{x^4} + \sqrt[3]{8x}$ (b) $4\sqrt{18rt^3} + 5\sqrt{32r^3t^5}$

53. (a) $\sqrt{81x^2 + 81}$ (b) $\sqrt{36x^2 + 36y^2}$

54. (a) $\sqrt{27a^2 + 63a}$ (b) $\sqrt{75t + 100t^2}$

55–60 ■ Rational Exponents Evaluate each expression.

55. (a) $16^{1/4}$ (b) $-8^{1/3}$ (c) $9^{-1/2}$

56. (a) $27^{1/3}$ (b) $(-8)^{1/3}$ (c) $-\left(\frac{1}{8}\right)^{1/3}$

57. (a) $32^{2/5}$ (b) $\left(\frac{4}{9}\right)^{-1/2}$ (c) $\left(\frac{16}{81}\right)^{3/4}$

58. (a) $125^{2/3}$ (b) $\left(\frac{25}{64}\right)^{3/2}$ (c) $27^{-4/3}$

59. (a) $5^{2/3}\cdot 5^{1/3}$ (b) $\dfrac{3^{3/5}}{3^{2/5}}$ (c) $(\sqrt[3]{4})^3$

60. (a) $3^{2/7}\cdot 3^{12/7}$ (b) $\dfrac{7^{2/3}}{7^{5/3}}$ (c) $(\sqrt[5]{6})^{-10}$

61–70 ■ Rational Exponents Simplify the expression and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

61. (a) $x^{3/4}x^{5/4}$ (b) $y^{2/3}y^{4/3}$

62. (a) $(4b)^{1/2}(8b^{1/4})$ (b) $(3a^{3/4})^2(5a^{1/2})$

63. (a) $\dfrac{w^{4/3}w^{2/3}}{w^{1/3}}$ (b) $\dfrac{a^{5/4}(2a^{3/4})^3}{a^{1/4}}$

64. (a) $(8y^3)^{-2/3}$ (b) $(u^4v^6)^{-1/3}$

65. (a) $(8a^6b^{3/2})^{2/3}$ (b) $(4a^6b^8)^{3/2}$

66. (a) $(x^{-5}y^{1/3})^{-3/5}$ (b) $(4r^8s^{-1/2})^{1/2}(32s^{-5/4})^{-1/5}$

67. (a) $\dfrac{(8s^3t^3)^{2/3}}{(s^4t^{-8})^{1/4}}$ (b) $\dfrac{(32x^5y^{-3/2})^{2/5}}{(x^{5/3}y^{2/3})^{3/5}}$

68. (a) $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4}$ (b) $\left(\dfrac{4s^3t^4}{s^2t^{9/2}}\right)^{-1/2}$

69. (a) $\left(\dfrac{x^{3/2}}{y^{-1/2}}\right)^4\left(\dfrac{x^{-2}}{y^3}\right)$ (b) $\left(\dfrac{4y^3z^{2/3}}{x^{1/2}}\right)^2\left(\dfrac{x^{-3}y^6}{8z^4}\right)^{1/3}$

70. (a) $\left(\dfrac{a^{1/6}b^{-3}}{x^{-1}y}\right)^3\left(\dfrac{x^{-2}b^{-1}}{a^{3/2}y^{1/3}}\right)$ (b) $\dfrac{(9st)^{3/2}}{(27s^3t^{-4})^{2/3}}\left(\dfrac{3s^{-2}}{4t^{1/3}}\right)^{-1}$

71–78 ■ Radicals Simplify the expression, and eliminate any negative exponents(s). Assume that all letters denote positive numbers.

71. (a) $\sqrt{x^3}$ (b) $\sqrt[5]{x^6}$

72. (a) $\sqrt{x^5}$ (b) $\sqrt[4]{x^6}$

73. (a) $\sqrt[6]{y^5}\sqrt[3]{y^2}$ (b) $(5\sqrt[3]{x})(2\sqrt[4]{x})$

74. (a) $\sqrt[4]{b^3}\sqrt{b}$ (b) $(2\sqrt{a})(\sqrt[3]{a^2})$

75. (a) $\sqrt{4st^3}\sqrt[6]{s^3t^2}$ (b) $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}}$

76. (a) $\sqrt[5]{x^3y^2}\sqrt[10]{x^4y^{16}}$ (b) $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}}$

77. (a) $\sqrt[3]{y\sqrt{y}}$ (b) $\sqrt{\dfrac{16u^3v}{uv^5}}$

78. (a) $\sqrt{s\sqrt{s^3}}$ (b) $\sqrt[3]{\dfrac{54x^2y^4}{2x^5y}}$

79–82 ■ Rationalize Put each fractional expression into standard form by rationalizing the denominator.

79. (a) $\dfrac{1}{\sqrt{6}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\dfrac{9}{\sqrt[4]{2}}$

80. (a) $\dfrac{12}{\sqrt{3}}$ (b) $\sqrt{\frac{12}{5}}$ (c) $\dfrac{8}{\sqrt[3]{5^2}}$

81. (a) $\dfrac{1}{\sqrt{5x}}$ (b) $\sqrt{\dfrac{x}{5}}$ (c) $\sqrt[5]{\dfrac{1}{x^3}}$

82. (a) $\sqrt{\dfrac{s}{3t}}$ (b) $\dfrac{a}{\sqrt[6]{b^2}}$ (c) $\dfrac{1}{c^{3/5}}$

83–84 ■ Scientific Notation Write each number in scientific notation.

83. (a) 69,300,000 (b) 7,200,000,000,000
(c) 0.000028536 (d) 0.0001213

84. (a) 129,540,000 (b) 7,259,000,000
(c) 0.0000000014 (d) 0.0007029

85–86 ■ Decimal Notation Write each number in decimal notation.

85. (a) 3.19×10^5 (b) 2.721×10^8
(c) 2.670×10^{-8} (d) 9.999×10^{-9}

86. (a) 7.1×10^{14} (b) 6×10^{12}
(c) 8.55×10^{-3} (d) 6.257×10^{-10}

87–88 ■ Scientific Notation Write the number indicated in each statement in scientific notation.

87. (a) A light-year, the distance that light travels in one year, is about 5,900,000,000,000 mi.
(b) The diameter of an electron is about 0.0000000000004 cm.
(c) A drop of water contains more than 33 billion billion molecules.

88. (a) The distance from the earth to the sun is about 93 million miles.
(b) The mass of an oxygen molecule is about 0.000000000000000000000053 g.
(c) The mass of the earth is about 5,970,000,000,000,000,000,000,000 kg.

89–94 ■ Scientific Notation Use scientific notation, the Laws of Exponents, and a calculator to perform the indicated operations. State your answer rounded to the number of significant digits indicated by the given data.

89. $(7.2 \times 10^{-9})(1.806 \times 10^{-12})$

90. $(1.062 \times 10^{24})(8.61 \times 10^{19})$

91. $\dfrac{1.295643 \times 10^9}{(3.610 \times 10^{-17})(2.511 \times 10^6)}$

92. $\dfrac{(73.1)(1.6341 \times 10^{28})}{0.0000000019}$

93. $\dfrac{(0.0000162)(0.01582)}{(594{,}621{,}000)(0.0058)}$ **94.** $\dfrac{(3.542 \times 10^{-6})^9}{(5.05 \times 10^4)^{12}}$

SKILLS Plus

95. Let a, b, and c be real numbers with $a > 0$, $b < 0$, and $c < 0$. Determine the sign of each expression.
(a) b^5 (b) b^{10} (c) ab^2c^3
(d) $(b - a)^3$ (e) $(b - a)^4$ (f) $\dfrac{a^3c^3}{b^6c^6}$

96. Comparing Roots Without using a calculator, determine which number is larger in each pair.
(a) $2^{1/2}$ or $2^{1/3}$ (b) $\left(\frac{1}{2}\right)^{1/2}$ or $\left(\frac{1}{2}\right)^{1/3}$
(c) $7^{1/4}$ or $4^{1/3}$ (d) $\sqrt[3]{5}$ or $\sqrt{3}$

APPLICATIONS

97. **Distance to the Nearest Star** Proxima Centauri, the star nearest to our solar system, is 4.3 light-years away. Use the information in Exercise 87(a) to express this distance in miles.

98. **Speed of Light** The speed of light is about 186,000 mi/s. Use the information in Exercise 88(a) to find how long it takes for a light ray from the sun to reach the earth.

99. **Volume of the Oceans** The average ocean depth is 3.7×10^3 m, and the area of the oceans is 3.6×10^{14} m^2. What is the total volume of the ocean in liters? (One cubic meter contains 1000 liters.)

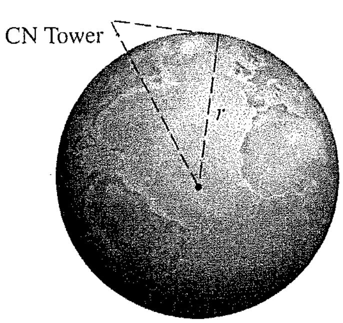
100. **National Debt** As of July 2013, the population of the United States was 3.164×10^8, and the national debt was 1.674×10^{13} dollars. How much was each person's share of the debt?
[*Source:* U.S. Census Bureau and U.S. Department of Treasury]

101. **Number of Molecules** A sealed room in a hospital, measuring 5 m wide, 10 m long, and 3 m high, is filled with pure oxygen. One cubic meter contains 1000 L, and 22.4 L of any gas contains 6.02×10^{23} molecules (Avogadro's number). How many molecules of oxygen are there in the room?

102. **How Far Can You See?** Because of the curvature of the earth, the maximum distance D that you can see from the top of a tall building of height h is estimated by the formula

$$D = \sqrt{2rh + h^2}$$

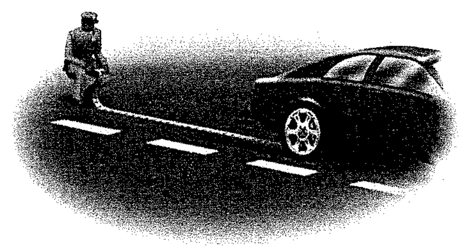
where $r = 3960$ mi is the radius of the earth and D and h are also measured in miles. How far can you see from the observation deck of the Toronto CN Tower, 1135 ft above the ground?



103. **Speed of a Skidding Car** Police use the formula $s = \sqrt{30fd}$ to estimate the speed s (in mi/h) at which a car is traveling if it skids d feet after the brakes are applied suddenly. The number f is the coefficient of friction of the road, which is a measure of the "slipperiness" of the road. The table gives some typical estimates for f.

	Tar	Concrete	Gravel
Dry	1.0	0.8	0.2
Wet	0.5	0.4	0.1

(a) If a car skids 65 ft on wet concrete, how fast was it moving when the brakes were applied?
(b) If a car is traveling at 50 mi/h, how far will it skid on wet tar?



104. **Distance from the Earth to the Sun** It follows from **Kepler's Third Law** of planetary motion that the average distance from a planet to the sun (in meters) is

$$d = \left(\frac{GM}{4\pi^2}\right)^{1/3} T^{2/3}$$

where $M = 1.99 \times 10^{30}$ kg is the mass of the sun, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2 is the gravitational constant, and T is the period of the planet's orbit (in seconds). Use the fact that the period of the earth's orbit is about 365.25 days to find the distance from the earth to the sun.

DISCUSS DISCOVER PROVE WRITE

105. **DISCUSS: How Big is a Billion?** If you had a million (10^6) dollars in a suitcase, and you spent a thousand (10^3) dollars each day, how many years would it take you to use all the money? Spending at the same rate, how many years would it take you to empty a suitcase filled with a *billion* (10^9) dollars?

106. **DISCUSS: Easy Powers that Look Hard** Calculate these expressions in your head. Use the Laws of Exponents to help you.

(a) $\dfrac{18^5}{9^5}$ (b) $20^6 \cdot (0.5)^6$

107. DISCOVER: Limiting Behavior of Powers Complete the following tables. What happens to the nth root of 2 as n gets large? What about the nth root of $\frac{1}{2}$?

n	$2^{1/n}$
1	
2	
5	
10	
100	

n	$\left(\frac{1}{2}\right)^{1/n}$
1	
2	
5	
10	
100	

Construct a similar table for $n^{1/n}$. What happens to the nth root of n as n gets large?

108. PROVE: Laws of Exponents Prove the following Laws of Exponents for the case in which m and n are positive integers and $m > n$.

(a) Law 2: $\dfrac{a^m}{a^n} = a^{m-n}$ **(b)** Law 5: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

109. PROVE: Laws of Exponents Prove the following Laws of Exponents.

(a) Law 6: $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$ **(b)** Law 7: $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$

1.3 ALGEBRAIC EXPRESSIONS

Adding and Subtracting Polynomials ▪ Multiplying Algebraic Expressions ▪ Special Product Formulas ▪ Factoring Common Factors ▪ Factoring Trinomials ▪ Special Factoring Formulas ▪ Factoring by Grouping Terms

A **variable** is a letter that can represent any number from a given set of numbers. If we start with variables, such as x, y, and z, and some real numbers and combine them using addition, subtraction, multiplication, division, powers, and roots, we obtain an **algebraic expression**. Here are some examples:

$$2x^2 - 3x + 4 \qquad \sqrt{x} + 10 \qquad \frac{y - 2z}{y^2 + 4}$$

A **monomial** is an expression of the form ax^k, where a is a real number and k is a nonnegative integer. A **binomial** is a sum of two monomials and a **trinomial** is a sum of three monomials. In general, a sum of monomials is called a *polynomial*. For example, the first expression listed above is a polynomial, but the other two are not.

POLYNOMIALS

A **polynomial** in the variable x is an expression of the form

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_0, a_1, \ldots, a_n$ are real numbers, and n is a nonnegative integer. If $a_n \neq 0$, then the polynomial has **degree n**. The monomials $a_k x^k$ that make up the polynomial are called the **terms** of the polynomial.

Note that the degree of a polynomial is the highest power of the variable that appears in the polynomial.

Polynomial	Type	Terms	Degree
$2x^2 - 3x + 4$	trinomial	$2x^2, -3x, 4$	2
$x^8 + 5x$	binomial	$x^8, 5x$	8
$8 - x + x^2 - \frac{1}{2}x^3$	four terms	$-\frac{1}{2}x^3, x^2, -x, 3$	3
$5x + 1$	binomial	$5x, 1$	1
$9x^5$	monomial	$9x^5$	5
6	monomial	6	0

Adding and Subtracting Polynomials

Distributive Property

$ac + bc = (a + b)c$

We **add** and **subtract** polynomials using the properties of real numbers that were discussed in Section 1.1. The idea is to combine **like terms** (that is, terms with the same variables raised to the same powers) using the Distributive Property. For instance,

$$5x^7 + 3x^7 = (5 + 3)x^7 = 8x^7$$

In subtracting polynomials, we have to remember that if a minus sign precedes an expression in parentheses, then the sign of every term within the parentheses is changed when we remove the parentheses:

$$-(b + c) = -b - c$$

[This is simply a case of the Distributive Property, $a(b + c) = ab + ac$, with $a = -1$.]

EXAMPLE 1 Adding and Subtracting Polynomials

(a) Find the sum $(x^3 - 6x^2 + 2x + 4) + (x^3 + 5x^2 - 7x)$.

(b) Find the difference $(x^3 - 6x^2 + 2x + 4) - (x^3 + 5x^2 - 7x)$.

SOLUTION

(a) $(x^3 - 6x^2 + 2x + 4) + (x^3 + 5x^2 - 7x)$

$= (x^3 + x^3) + (-6x^2 + 5x^2) + (2x - 7x) + 4$ Group like terms

$= 2x^3 - x^2 - 5x + 4$ Combine like terms

(b) $(x^3 - 6x^2 + 2x + 4) - (x^3 + 5x^2 - 7x)$

$= x^3 - 6x^2 + 2x + 4 - x^3 - 5x^2 + 7x$ Distributive Property

$= (x^3 - x^3) + (-6x^2 - 5x^2) + (2x + 7x) + 4$ Group like terms

$= -11x^2 + 9x + 4$ Combine like terms

Now Try Exercises 17 and 19

Multiplying Algebraic Expressions

To find the **product** of polynomials or other algebraic expressions, we need to use the Distributive Property repeatedly. In particular, using it three times on the product of two binomials, we get

$$(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$$

This says that we multiply the two factors by multiplying each term in one factor by each term in the other factor and adding these products. Schematically, we have

$$(a + b)(c + d) = \underset{\mathrm{F}}{ac} + \underset{\mathrm{O}}{ad} + \underset{\mathrm{I}}{bc} + \underset{\mathrm{L}}{bd}$$

The acronym **FOIL** helps us remember that the product of two binomials is the sum of the products of the **F**irst terms, the **O**uter terms, the **I**nner terms, and the **L**ast terms.

In general, we can multiply two algebraic expressions by using the Distributive Property and the Laws of Exponents.

EXAMPLE 2 Multiplying Binomials Using FOIL

$(2x + 1)(3x - 5) = \underset{\mathrm{F}}{6x^2} - \underset{\mathrm{O}}{10x} + \underset{\mathrm{I}}{3x} - \underset{\mathrm{L}}{5}$ Distributive Property

$= 6x^2 - 7x - 5$ Combine like terms

Now Try Exercise 25

When we multiply trinomials or other polynomials with more terms, we use the Distributive Property. It is also helpful to arrange our work in table form. The next example illustrates both methods.

EXAMPLE 3 Multiplying Polynomials

Find the product: $(2x + 3)(x^2 - 5x + 4)$

SOLUTION 1: Using the Distributive Property

$$
\begin{aligned}
(2x + 3)(x^2 - 5x + 4) &= 2x(x^2 - 5x + 4) + 3(x^2 - 5x + 4) && \text{Distributive Property} \\
&= (2x \cdot x^2 - 2x \cdot 5x + 2x \cdot 4) + (3 \cdot x^2 - 3 \cdot 5x + 3 \cdot 4) && \text{Distributive Property} \\
&= (2x^3 - 10x^2 + 8x) + (3x^2 - 15x + 12) && \text{Laws of Exponents} \\
&= 2x^3 - 7x^2 - 7x + 12 && \text{Combine like terms}
\end{aligned}
$$

SOLUTION 2: Using Table Form

$$
\begin{array}{rrrrl}
 & x^2 & - 5x & + 4 & \\
 & & 2x & + 3 & \\
\hline
 & 3x^2 & - 15x & + 12 & \text{Multiply } x^2 - 5x + 4 \text{ by } 3 \\
2x^3 & - 10x^2 & + 8x & & \text{Multiply } x^2 - 5x + 4 \text{ by } 2x \\
\hline
2x^3 & - 7x^2 & - 7x & + 12 & \text{Add like terms}
\end{array}
$$

Now Try Exercise 47

Special Product Formulas

Certain types of products occur so frequently that you should memorize them. You can verify the following formulas by performing the multiplications.

SPECIAL PRODUCT FORMULAS

If A and B are any real numbers or algebraic expressions, then

1. $(A + B)(A - B) = A^2 - B^2$	Sum and difference of same terms
2. $(A + B)^2 = A^2 + 2AB + B^2$	Square of a sum
3. $(A - B)^2 = A^2 - 2AB + B^2$	Square of a difference
4. $(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$	Cube of a sum
5. $(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$	Cube of a difference

The key idea in using these formulas (or any other formula in algebra) is the **Principle of Substitution**: We may substitute any algebraic expression for any letter in a formula. For example, to find $(x^2 + y^3)^2$ we use Product Formula 2, substituting x^2 for A and y^3 for B, to get

$$(x^2 + y^3)^2 = (x^2)^2 + 2(x^2)(y^3) + (y^3)^2$$

$$(A + B)^2 = A^2 + 2AB + B^2$$

Mathematics in the Modern World

Changing Words, Sound, and Pictures into Numbers

Pictures, sound, and text are routinely transmitted from one place to another via the Internet, fax machines, or modems. How can such things be transmitted through telephone wires? The key to doing this is to change them into numbers or bits (the digits 0 or 1). It's easy to see how to change text to numbers. For example, we could use the correspondence A = 00000001, B = 00000010, C = 00000011, D = 00000100, E = 00000101, and so on. The word "BED" then becomes 000000100000010100000100. By reading the digits in groups of eight, it is possible to translate this number back to the word "BED."

Changing sound to bits is more complicated. A sound wave can be graphed on an oscilloscope or a computer. The graph is then broken down mathematically into simpler components corresponding to the different frequencies of the original sound. (A branch of mathematics called Fourier analysis is used here.) The intensity of each component is a number, and the original sound can be reconstructed from these numbers. For example, music is stored on a CD as a sequence of bits; it may look like 101010001010010100101010 10000010 11110101000101011.... (One second of music requires 1.5 million bits!) The CD player reconstructs the music from the numbers on the CD.

Changing pictures into numbers involves expressing the color and brightness of each dot (or pixel) into a number. This is done very efficiently using a branch of mathematics called wavelet theory. The FBI uses wavelets as a compact way to store the millions of fingerprints they need on file.

EXAMPLE 4 Using the Special Product Formulas

Use the Special Product Formulas to find each product.

(a) $(3x + 5)^2$ **(b)** $(x^2 - 2)^3$

SOLUTION

(a) Substituting $A = 3x$ and $B = 5$ in Product Formula 2, we get

$$(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2 = 9x^2 + 30x + 25$$

(b) Substituting $A = x^2$ and $B = 2$ in Product Formula 5, we get

$$\begin{aligned}(x^2 - 2)^3 &= (x^2)^3 - 3(x^2)^2(2) + 3(x^2)(2)^2 - 2^3\\ &= x^6 - 6x^4 + 12x^2 - 8\end{aligned}$$

Now Try Exercises 31 and 43

EXAMPLE 5 Using the Special Product Formulas

Find each product.

(a) $(2x - \sqrt{y})(2x + \sqrt{y})$ **(b)** $(x + y - 1)(x + y + 1)$

SOLUTION

(a) Substituting $A = 2x$ and $B = \sqrt{y}$ in Product Formula 1, we get

$$(2x - \sqrt{y})(2x + \sqrt{y}) = (2x)^2 - (\sqrt{y})^2 = 4x^2 - y$$

(b) If we group $x + y$ together and think of this as one algebraic expression, we can use Product Formula 1 with $A = x + y$ and $B = 1$.

$$\begin{aligned}(x + y - 1)(x + y + 1) &= [(x + y) - 1][(x + y) + 1]\\ &= (x + y)^2 - 1^2 && \text{Product Formula 1}\\ &= x^2 + 2xy + y^2 - 1 && \text{Product Formula 2}\end{aligned}$$

Now Try Exercises 57 and 61

Factoring Common Factors

We use the Distributive Property to expand algebraic expressions. We sometimes need to reverse this process (again using the Distributive Property) by **factoring** an expression as a product of simpler ones. For example, we can write

FACTORING →

$$x^2 - 4 = (x - 2)(x + 2)$$

← EXPANDING

We say that $x - 2$ and $x + 2$ are **factors** of $x^2 - 4$.

The easiest type of factoring occurs when the terms have a common factor.

EXAMPLE 6 Factoring Out Common Factors

Factor each expression.

(a) $3x^2 - 6x$ **(b)** $8x^4y^2 + 6x^3y^3 - 2xy^4$ **(c)** $(2x + 4)(x - 3) - 5(x - 3)$

SOLUTION

(a) The greatest common factor of the terms $3x^2$ and $-6x$ is $3x$, so we have

$$3x^2 - 6x = 3x(x - 2)$$

CHECK YOUR ANSWER

Multiplying gives

$$3x(x - 2) = 3x^2 - 6x \checkmark$$

(b) We note that

$$8, 6, \text{ and } -2 \text{ have the greatest common factor } 2$$
$$x^4, x^3, \text{ and } x \text{ have the greatest common factor } x$$
$$y^2, y^3, \text{ and } y^4 \text{ have the greatest common factor } y^2$$

So the greatest common factor of the three terms in the polynomial is $2xy^2$, and we have

$$8x^4y^2 + 6x^3y^3 - 2xy^4 = (2xy^2)(4x^3) + (2xy^2)(3x^2y) + (2xy^2)(-y^2)$$
$$= 2xy^2(4x^3 + 3x^2y - y^2)$$

CHECK YOUR ANSWER

Multiplying gives

$$2xy^2(4x^3 + 3x^2y - y^2) = 8x^4y^2 + 6x^3y^3 - 2xy^4 \quad ✓$$

(c) The two terms have the common factor $x - 3$.

$$(2x + 4)(x - 3) - 5(x - 3) = [(2x + 4) - 5](x - 3) \quad \text{Distributive Property}$$
$$= (2x - 1)(x - 3) \quad \text{Simplify}$$

Now Try Exercises 63, 65, and 67 ■

Factoring Trinomials

To factor a trinomial of the form $x^2 + bx + c$, we note that

$$(x + r)(x + s) = x^2 + (r + s)x + rs$$

so we need to choose numbers r and s so that $r + s = b$ and $rs = c$.

EXAMPLE 7 Factoring $x^2 + bx + c$ by Trial and Error

Factor: $x^2 + 7x + 12$

SOLUTION We need to find two integers whose product is 12 and whose sum is 7. By trial and error we find that the two integers are 3 and 4. Thus the factorization is

$$x^2 + 7x + 12 = (x + \underbrace{3)(x + 4}_{\text{factors of 12}})$$

CHECK YOUR ANSWER

Multiplying gives

$$(x + 3)(x + 4) = x^2 + 7x + 12 \quad ✓$$

Now Try Exercise 69 ■

To factor a trinomial of the form $ax^2 + bx + c$ with $a \neq 1$, we look for factors of the form $px + r$ and $qx + s$:

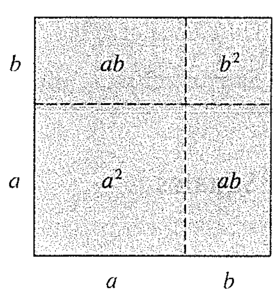
$$ax^2 + bx + c = (px + r)(qx + s) = pqx^2 + (ps + qr)x + rs$$

Therefore we try to find numbers p, q, r, and s such that $pq = a$, $rs = c$, $ps + qr = b$. If these numbers are all integers, then we will have a limited number of possibilities to try for p, q, r, and s.

factors of a

$$ax^2 + bx + c = (px + r)(qx + s)$$

factors of c



DISCOVERY PROJECT

Visualizing a Formula

Many of the Special Product Formulas in this section can be "seen" as geometrical facts about length, area, and volume. For example, the formula about the square of a sum can be interpreted to be about areas of squares and rectangles. The ancient Greeks always interpreted algebraic formulas in terms of geometric figures. Such figures give us special insight into how these formulas work. You can find the project at **www.stewartmath.com**.

EXAMPLE 8 Factoring $ax^2 + bx + c$ by Trial and Error

Factor: $6x^2 + 7x - 5$

SOLUTION We can factor 6 as $6 \cdot 1$ or $3 \cdot 2$, and -5 as $-5 \cdot 1$ or $5 \cdot (-1)$. By trying these possibilities, we arrive at the factorization

factors of 6

$$6x^2 + 7x - 5 = (3x + 5)(2x - 1)$$

factors of −5

CHECK YOUR ANSWER

Multiplying gives

$(3x + 5)(2x - 1) = 6x^2 + 7x - 5$ ✓

Now Try Exercise 71

EXAMPLE 9 Recognizing the Form of an Expression

Factor each expression.

(a) $x^2 - 2x - 3$ **(b)** $(5a + 1)^2 - 2(5a + 1) - 3$

SOLUTION

(a) $x^2 - 2x - 3 = (x - 3)(x + 1)$ Trial and error

(b) This expression is of the form

$$\quad^2 - 2 \quad - 3$$

where represents $5a + 1$. This is the same form as the expression in part (a), so it will factor as ($- 3$)($+ 1$).

$$(5a + 1)^2 - 2(5a + 1) - 3 = [(5a + 1) - 3][(5a + 1) + 1]$$
$$= (5a - 2)(5a + 2)$$

Now Try Exercise 75

Special Factoring Formulas

Some special algebraic expressions can be factored by using the following formulas. The first three are simply Special Product Formulas written backward.

SPECIAL FACTORING FORMULAS

Formula	Name
1. $A^2 - B^2 = (A - B)(A + B)$	Difference of squares
2. $A^2 + 2AB + B^2 = (A + B)^2$	Perfect square
3. $A^2 - 2AB + B^2 = (A - B)^2$	Perfect square
4. $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$	Difference of cubes
5. $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$	Sum of cubes

EXAMPLE 10 Factoring Differences of Squares

Factor each expression.

(a) $4x^2 - 25$ **(b)** $(x + y)^2 - z^2$

Terms and Factors

When we multiply two numbers together, each of the numbers is called a **factor** of the product. When we add two numbers together, each number is called a **term** of the sum.

2×3	$2 + 3$
Factors	Terms

If a factor is common to each term of an expression we can factor it out. The following expression has two terms.

$$ax + 2ay$$

a is a factor of each term

Each term contains the factor a, so we can factor a out and write the expression as

$$ax + 2ay = a(x + 2y)$$

SOLUTION

(a) Using the Difference of Squares Formula with $A = 2x$ and $B = 5$, we have

$$4x^2 - 25 = (2x)^2 - 5^2 = (2x - 5)(2x + 5)$$

$$A^2 - B^2 = (A - B)(A + B)$$

(b) We use the Difference of Squares Formula with $A = x + y$ and $B = z$.

$$(x + y)^2 - z^2 = (x + y - z)(x + y + z)$$

Now Try Exercises 77 and 111 ■

A trinomial is a perfect square if it is of the form

$$A^2 + 2AB + B^2 \qquad \text{or} \qquad A^2 - 2AB + B^2$$

So we **recognize a perfect square** if the middle term ($2AB$ or $-2AB$) is plus or minus twice the product of the square roots of the outer two terms.

EXAMPLE 11 Recognizing Perfect Squares

Factor each trinomial.

(a) $x^2 + 6x + 9$ **(b)** $4x^2 - 4xy + y^2$

SOLUTION

(a) Here $A = x$ and $B = 3$, so $2AB = 2 \cdot x \cdot 3 = 6x$. Since the middle term is $6x$, the trinomial is a perfect square. By the Perfect Square Formula we have

$$x^2 + 6x + 9 = (x + 3)^2$$

(b) Here $A = 2x$ and $B = y$, so $2AB = 2 \cdot 2x \cdot y = 4xy$. Since the middle term is $-4xy$, the trinomial is a perfect square. By the Perfect Square Formula we have

$$4x^2 - 4xy + y^2 = (2x - y)^2$$

Now Try Exercises 107 and 109 ■

EXAMPLE 12 Factoring Differences and Sums of Cubes

Factor each polynomial.

(a) $27x^3 - 1$ **(b)** $x^6 + 8$

SOLUTION

(a) Using the Difference of Cubes Formula with $A = 3x$ and $B = 1$, we get

$$27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)[(3x)^2 + (3x)(1) + 1^2]$$
$$= (3x - 1)(9x^2 + 3x + 1)$$

(b) Using the Sum of Cubes Formula with $A = x^2$ and $B = 2$, we have

$$x^6 + 8 = (x^2)^3 + 2^3 = (x^2 + 2)(x^4 - 2x^2 + 4)$$

Now Try Exercises 79 and 81 ■

When we factor an expression, the result can sometimes be factored further. In general, *we first factor out common factors*, then inspect the result to see whether it can be factored by any of the other methods of this section. We repeat this process until we have factored the expression completely.

EXAMPLE 13 Factoring an Expression Completely

Factor each expression completely.

(a) $2x^4 - 8x^2$ **(b)** $x^5y^2 - xy^6$

SOLUTION

(a) We first factor out the power of x with the smallest exponent.

$$\begin{aligned} 2x^4 - 8x^2 &= 2x^2(x^2 - 4) && \text{Common factor is } 2x^2 \\ &= 2x^2(x - 2)(x + 2) && \text{Factor } x^2 - 4 \text{ as a difference of squares} \end{aligned}$$

(b) We first factor out the powers of x and y with the smallest exponents.

$$\begin{aligned} x^5y^2 - xy^6 &= xy^2(x^4 - y^4) && \text{Common factor is } xy^2 \\ &= xy^2(x^2 + y^2)(x^2 - y^2) && \text{Factor } x^4 - y^4 \text{ as a difference of squares} \\ &= xy^2(x^2 + y^2)(x + y)(x - y) && \text{Factor } x^2 - y^2 \text{ as a difference of squares} \end{aligned}$$

Now Try Exercises 117 and 119

In the next example we factor out variables with fractional exponents. This type of factoring occurs in calculus.

EXAMPLE 14 Factoring Expressions with Fractional Exponents

Factor each expression.

(a) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ **(b)** $(2 + x)^{-2/3}x + (2 + x)^{1/3}$

SOLUTION

(a) Factor out the power of x with the *smallest exponent*, that is, $x^{-1/2}$.

$$\begin{aligned} 3x^{3/2} - 9x^{1/2} + 6x^{-1/2} &= 3x^{-1/2}(x^2 - 3x + 2) && \text{Factor out } 3x^{-1/2} \\ &= 3x^{-1/2}(x - 1)(x - 2) && \text{Factor the quadratic } x^2 - 3x + 2 \end{aligned}$$

To factor out $x^{-1/2}$ from $x^{3/2}$, we *subtract* exponents:

$$\begin{aligned} x^{3/2} &= x^{-1/2}(x^{3/2-(-1/2)}) \\ &= x^{-1/2}(x^{3/2+1/2}) \\ &= x^{-1/2}(x^2) \end{aligned}$$

(b) Factor out the power of $2 + x$ with the *smallest exponent*, that is, $(2 + x)^{-2/3}$.

$$\begin{aligned} (2 + x)^{-2/3}x + (2 + x)^{1/3} &= (2 + x)^{-2/3}[x + (2 + x)] && \text{Factor out } (2 + x)^{-2/3} \\ &= (2 + x)^{-2/3}(2 + 2x) && \text{Simplify} \\ &= 2(2 + x)^{-2/3}(1 + x) && \text{Factor out 2} \end{aligned}$$

CHECK YOUR ANSWERS

To see that you have factored correctly, multiply using the Laws of Exponents.

(a) $3x^{-1/2}(x^2 - 3x + 2)$
$= 3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ ✓

(b) $(2 + x)^{-2/3}[x + (2 + x)]$
$= (2 + x)^{-2/3}x + (2 + x)^{1/3}$ ✓

Now Try Exercises 93 and 95

Factoring by Grouping Terms

Polynomials with at least four terms can sometimes be factored by grouping terms. The following example illustrates the idea.

EXAMPLE 15 Factoring by Grouping

Factor each polynomial.

(a) $x^3 + x^2 + 4x + 4$ **(b)** $x^3 - 2x^2 - 9x + 18$

SOLUTION

(a) $x^3 + x^2 + 4x + 4 = (x^3 + x^2) + (4x + 4)$ Group terms

$= x^2(x + 1) + 4(x + 1)$ Factor out common factors

$= (x^2 + 4)(x + 1)$ Factor $x + 1$ from each term

(b) $x^3 - 2x^2 - 9x + 18 = (x^3 - 2x^2) - (9x - 18)$ Group terms

$= x^2(x - 2) - 9(x - 2)$ Factor common factors

$= (x^2 - 9)(x - 2)$ Factor $(x - 2)$ from each term

$= (x - 3)(x + 3)(x - 2)$ Factor completely

Now Try Exercises 85 and 121

1.3 EXERCISES

CONCEPTS

1. Consider the polynomial $2x^5 + 6x^4 + 4x^3$.
   (a) How many terms does this polynomial have? ________
   List the terms: ________________.
   (b) What factor is common to each term? ________
   Factor the polynomial: $2x^5 + 6x^4 + 4x^3$ = ________.
2. To factor the trinomial $x^2 + 7x + 10$, we look for two integers whose product is ________ and whose sum is ________. These integers are ________ and ________, so the trinomial factors as ________________.
3. The greatest common factor in the expression $3x^3 + x^2$ is ________, and the expression factors as (+).
4. The Special Product Formula for the "square of a sum" is $(A + B)^2$ = ________________.
   So $(2x + 3)^2$ = ________________.
5. The Special Product Formula for the "product of the sum and difference of terms" is $(A + B)(A - B)$ = ________________.
   So $(5 + x)(5 - x)$ = ________________.
6. The Special Factoring Formula for the "difference of squares" is $A^2 - B^2$ = ________________. So $4x^2 - 25$ factors as ________________.
7. The Special Factoring Formula for a "perfect square" is $A^2 + 2AB + B^2$ = ________________. So $x^2 + 10x + 25$ factors as ________________.
8. *Yes or No*? If *No*, give a reason.
   (a) Is the expression $(x + 5)^2$ equal to $x^2 + 25$?
   (b) When you expand $(x + a)^2$, where $a \neq 0$, do you get three terms?
   (c) Is the expression $(x + 5)(x - 5)$ equal to $x^2 - 25$?
   (d) When you expand $(x + a)(x - a)$, where $a \neq 0$, do you get two terms?

SKILLS

9–14 ■ Polynomials Complete the following table by stating whether the polynomial is a monomial, binomial, or trinomial; then list its terms and state its degree.

Polynomial	Type	Terms	Degree
9. $5x^3 + 6$			
10. $-2x^2 + 5x - 3$			
11. -8			
12. $\frac{1}{2}x^7$			
13. $x - x^2 + x^3 - x^4$			
14. $\sqrt{2}x - \sqrt{3}$			

15–24 ■ Polynomials Find the sum, difference, or product.

15. $(12x - 7) - (5x - 12)$
16. $(5 - 3x) + (2x - 8)$
17. $(-2x^2 - 3x + 1) + (3x^2 + 5x - 4)$
18. $(3x^2 + x + 1) - (2x^2 - 3x - 5)$
19. $(5x^3 + 4x^2 - 3x) - (x^2 + 7x + 2)$
20. $3(x - 1) + 4(x + 2)$
21. $8(2x + 5) - 7(x - 9)$
22. $4(x^2 - 3x + 5) - 3(x^2 - 2x + 1)$
23. $2(2 - 5t) + t^2(t - 1) - (t^4 - 1)$
24. $5(3t - 4) - (t^2 + 2) - 2t(t - 3)$

25–30 ■ Using FOIL Multiply the algebraic expressions using the FOIL method and simplify.

25. $(3t - 2)(7t - 4)$
26. $(4s - 1)(2s + 5)$
27. $(3x + 5)(2x - 1)$
28. $(7y - 3)(2y - 1)$
29. $(x + 3y)(2x - y)$
30. $(4x - 5y)(3x - y)$

31–46 ■ Using Special Product Formulas Multiply the algebraic expressions using a Special Product Formula and simplify.

31. $(5x + 1)^2$ **32.** $(2 - 7y)^2$

33. $(2u + v)^2$ **34.** $(x - 3y)^2$

35. $(2x + 3y)^2$ **36.** $(r - 2s)^2$

37. $(x + 6)(x - 6)$ **38.** $(5 - y)(5 + y)$

39. $(3x - 4)(3x + 4)$ **40.** $(2y + 5)(2y - 5)$

41. $(\sqrt{x} + 2)(\sqrt{x} - 2)$ **42.** $(\sqrt{y} + \sqrt{2})(\sqrt{y} - \sqrt{2})$

43. $(y + 2)^3$ **44.** $(x - 3)^3$

45. $(1 - 2r)^3$ **46.** $(3 + 2y)^3$

47–62 ■ Multiplying Algebraic Expressions Perform the indicated operations and simplify.

47. $(x + 2)(x^2 + 2x + 3)$ **48.** $(x + 1)(2x^2 - x + 1)$

49. $(2x - 5)(x^2 - x + 1)$ **50.** $(1 + 2x)(x^2 - 3x + 1)$

51. $\sqrt{x}(x - \sqrt{x})$ **52.** $x^{3/2}(\sqrt{x} - 1/\sqrt{x})$

53. $y^{1/3}(y^{2/3} + y^{5/3})$ **54.** $x^{1/4}(2x^{3/4} - x^{1/4})$

55. $(x^2 - a^2)(x^2 + a^2)$

56. $(x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})$

57. $(\sqrt{a} - b)(\sqrt{a} + b)$

58. $(\sqrt{h^2 + 1} + 1)(\sqrt{h^2 + 1} - 1)$

59. $((x - 1) + x^2)((x - 1) - x^2)$

60. $(x + (2 + x^2))(x - (2 + x^2))$

61. $(2x + y - 3)(2x + y + 3)$

62. $(x + y + z)(x - y - z)$

63–68 ■ Factoring Common Factor Factor out the common factor.

63. $-2x^3 + x$ **64.** $3x^4 - 6x^3 - x^2$

65. $y(y - 6) + 9(y - 6)$ **66.** $(z + 2)^2 - 5(z + 2)$

67. $2x^2y - 6xy^2 + 3xy$ **68.** $-7x^4y^2 + 14xy^3 + 21xy^4$

69–76 ■ Factoring Trinomials Factor the trinomial.

69. $x^2 + 8x + 7$ **70.** $x^2 + 4x - 5$

71. $8x^2 - 14x - 15$ **72.** $6y^2 + 11y - 21$

73. $3x^2 - 16x + 5$ **74.** $5x^2 - 7x - 6$

75. $(3x + 2)^2 + 8(3x + 2) + 12$

76. $2(a + b)^2 + 5(a + b) - 3$

77–84 ■ Using Special Factoring Formulas Use a Special Factoring Formula to factor the expression.

77. $9a^2 - 16$ **78.** $(x + 3)^2 - 4$

79. $27x^3 + y^3$ **80.** $a^3 - b^6$

81. $8s^3 - 125t^3$ **82.** $1 + 1000y^3$

83. $x^2 + 12x + 36$ **84.** $16z^2 - 24z + 9$

85–90 ■ Factoring by Grouping Factor the expression by grouping terms.

85. $x^3 + 4x^2 + x + 4$ **86.** $3x^3 - x^2 + 6x - 2$

87. $5x^3 + x^2 + 5x + 1$ **88.** $18x^3 + 9x^2 + 2x + 1$

89. $x^3 + x^2 + x + 1$ **90.** $x^5 + x^4 + x + 1$

91–96 ■ Fractional Exponents Factor the expression completely. Begin by factoring out the lowest power of each common factor.

91. $x^{5/2} - x^{1/2}$ **92.** $3x^{-1/2} + 4x^{1/2} + x^{3/2}$

93. $x^{-3/2} + 2x^{-1/2} + x^{1/2}$ **94.** $(x - 1)^{7/2} - (x - 1)^{3/2}$

95. $(x^2 + 1)^{1/2} + 2(x^2 + 1)^{-1/2}$

96. $x^{-1/2}(x + 1)^{1/2} + x^{1/2}(x + 1)^{-1/2}$

97–126 ■ Factoring Completely Factor the expression completely.

97. $12x^3 + 18x$ **98.** $30x^3 + 15x^4$

99. $x^2 - 2x - 8$ **100.** $x^2 - 14x + 48$

101. $2x^2 + 5x + 3$ **102.** $2x^2 + 7x - 4$

103. $9x^2 - 36x - 45$ **104.** $8x^2 + 10x + 3$

105. $49 - 4y^2$ **106.** $4t^2 - 9s^2$

107. $t^2 - 6t + 9$ **108.** $x^2 + 10x + 25$

109. $4x^2 + 4xy + y^2$ **110.** $r^2 - 6rs + 9s^2$

111. $(a + b)^2 - (a - b)^2$ **112.** $\left(1 + \frac{1}{x}\right)^2 - \left(1 - \frac{1}{x}\right)^2$

113. $x^2(x^2 - 1) - 9(x^2 - 1)$ **114.** $(a^2 - 1)b^2 - 4(a^2 - 1)$

115. $8x^3 - 125$ **116.** $x^6 + 64$

117. $x^3 + 2x^2 + x$ **118.** $3x^3 - 27x$

119. $x^4y^3 - x^2y^5$ **120.** $18y^3x^2 - 2xy^4$

121. $3x^3 - x^2 - 12x + 4$ **122.** $9x^3 + 18x^2 - x - 2$

123. $(x - 1)(x + 2)^2 - (x - 1)^2(x + 2)$

124. $y^4(y + 2)^3 + y^5(y + 2)^4$

125. $(a^2 + 1)^2 - 7(a^2 + 1) + 10$

126. $(a^2 + 2a)^2 - 2(a^2 + 2a) - 3$

127–130 ■ Factoring Completely Factor the expression completely. (This type of expression arises in calculus when using the "Product Rule.")

127. $5(x^2 + 4)^4(2x)(x - 2)^4 + (x^2 + 4)^5(4)(x - 2)^3$

128. $3(2x - 1)^2(2)(x + 3)^{1/2} + (2x - 1)^3(\frac{1}{2})(x + 3)^{-1/2}$

129. $(x^2 + 3)^{-1/3} - \frac{2}{3}x^2(x^2 + 3)^{-4/3}$

130. $\frac{1}{2}x^{-1/2}(3x + 4)^{1/2} - \frac{3}{2}x^{1/2}(3x + 4)^{-1/2}$

SKILLS Plus

131–132 ■ Verifying Identities Show that the following identities hold.

131. **(a)** $ab = \frac{1}{2}[(a + b)^2 - (a^2 + b^2)]$
(b) $(a^2 + b^2)^2 - (a^2 - b^2)^2 = 4a^2b^2$

ANSWERS to Exercises and Chapter Tests

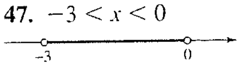
PROLOGUE ▸ PAGE P4

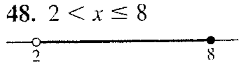
1. It can't go fast enough. **2.** 40% discount
3. 427, $3n+1$ **4.** 57 min **5.** No, not necessarily
6. The same amount **7.** 2π
8. The North Pole is one such point; there are infinitely many others near the South Pole.

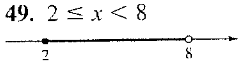
CHAPTER 1

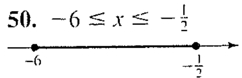
SECTION 1.1 ▸ PAGE 10

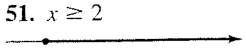
1. Answers may vary. Examples: **(a)** 2 **(b)** -3 **(c)** $\frac{3}{2}$ **(d)** $\sqrt{2}$ **2. (a)** ba; Commutative **(b)** $(a+b)+c$; Associative **(c)** $ab+ac$; Distributive **3. (a)** $\{x \mid 2 < x < 7\}$ **(b)** $(2, 7)$ **4.** absolute value; positive **5.** $|b-a|$; 7
6. (a) Yes **(b)** No **7. (a)** No **(b)** No
8. (a) Yes **(b)** Yes **9. (a)** 100 **(b)** 0, 100, -8
(c) $-1.5, 0, \frac{5}{2}, 2.71, 3.1\overline{4}, 100, -8$ **(d)** $\sqrt{7}, -\pi$
10. (a) $\sqrt{16}$ **(b)** $-500, \sqrt{16}, -\frac{20}{5}$
(c) $1.3, 1.3333\ldots, 5.34, -500, 1\frac{2}{3}, \sqrt{16}, \frac{246}{579}, -\frac{20}{5}$ **(d)** $\sqrt{5}$
11. Commutative Property of Addition **12.** Commutative Property of Multiplication **13.** Associative Property of Addition **14.** Distributive Property **15.** Distributive Property **16.** Distributive Property **17.** Commutative Property of Multiplication **18.** Distributive Property **19.** $3+x$
20. $(7 \cdot 3)x$ **21.** $4A + 4B$ **22.** $5(x+y)$ **23.** $3x+3y$
24. $8a - 8b$ **25.** $8m$ **26.** $-8y$ **27.** $-5x + 10y$
28. $3ab + 3ac - 6ad$ **29. (a)** $\frac{17}{30}$ **(b)** $\frac{9}{20}$ **30. (a)** $\frac{1}{15}$
(b) $\frac{35}{24}$ **31. (a)** 3 **(b)** $\frac{13}{20}$ **32. (a)** $\frac{8}{3}$ **(b)** 3 **33. (a)** $<$
(b) $>$ **(c)** $=$ **34. (a)** $<$ **(b)** $>$ **(c)** $=$ **35. (a)** False
(b) True **36. (a)** False **(b)** False **37. (a)** True **(b)** False
38. (a) True **(b)** True **39. (a)** $x > 0$ **(b)** $t < 4$ **(c)** $a \ge \pi$
(d) $-5 < x < \frac{1}{3}$ **(e)** $|3-p| \le 5$ **40. (a)** $y < 0$
(b) $z > 1$ **(c)** $b \le 8$ **(d)** $0 < w \le 17$ **(e)** $|y - \pi| \ge 2$
41. (a) $\{1, 2, 3, 4, 5, 6, 7, 8\}$ **(b)** $\{2, 4, 6\}$
42. (a) $\{2, 4, 6, 7, 8, 9, 10\}$ **(b)** $\{8\}$
43. (a) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **(b)** $\{7\}$
44. (a) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ **(b)** $\varnothing$
45. (a) $\{x \mid x \le 5\}$ **(b)** $\{x \mid -1 < x < 4\}$
46. (a) $\{x \mid -1 < x \le 5\}$ **(b)** $\{x \mid -2 \le x < 4\}$

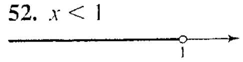
47. $-3 < x < 0$


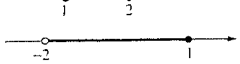
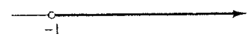
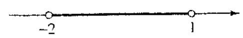
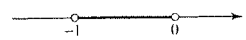
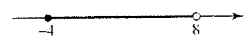
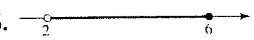
48. $2 < x \le 8$


49. $2 \le x < 8$


50. $-6 \le x \le -\frac{1}{2}$


51. $x \ge 2$


52. $x < 1$


53. $(-\infty, 1]$
54. $[1, 2]$ 
55. $(-2, 1]$ 
56. $[-5, \infty)$ 
57. $(-1, \infty)$ 
58. $(-5, 2)$ 
59. (a) $[-3, 5]$ **(b)** $(-3, 5]$ **60. (a)** $[0, 2)$ **(b)** $(-2, 0]$
61.  **62.** 
63.  **64.** 
65.  **66.** 

67. (a) 100 **(b)** 73 **68. (a)** $5 - \sqrt{5}$ **(b)** $10 - \pi$
69. (a) 2 **(b)** -1 **70. (a)** 10 **(b)** -1 **71. (a)** 12
(b) 5 **72. (a)** $\frac{1}{4}$ **(b)** 1 **73.** 5 **74.** 4 **75. (a)** 15 **(b)** 24
(c) $\frac{67}{40}$ **76. (a)** $\frac{18}{35}$ **(b)** 19 **(c)** 0.8 **77. (a)** $\frac{7}{9}$ **(b)** $\frac{13}{45}$
(c) $\frac{19}{33}$ **78. (a)** $\frac{518}{99}$ **(b)** $\frac{62}{45}$ **(c)** $\frac{1057}{495}$ **79.** $\pi - 3$
80. $\sqrt{2} - 1$ **81.** $b - a$ **82.** $2b$ **83. (a)** $-$ **(b)** $+$
(c) $+$ **(d)** $-$ **84. (a)** $+$ **(b)** $+$ **(c)** $-$ **(d)** $+$
85. Distributive Property
86. $T_O - T_G$: $-9, -3, 0, 5, 8, 1, -1$
$|T_O - T_G|$: 9, 3, 0, 5, 8, 1, 1
$T_O - T_G$ gives more information because it tells us which city had the higher (or lower) temperature.
87. (a) Yes, no **(b)** 6 ft

SECTION 1.2 ▸ PAGE 21

1. (a) 5^6 **(b)** base, exponent **2. (a)** add, 3^9 **(b)** subtract, 3^3
3. (a) $5^{1/3}$ **(b)** $\sqrt{5}$ **(c)** No **4.** $(4^{1/2})^3 = 8, (4^3)^{1/2} = 8$
5. $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ **6.** $\frac{2}{3}$ **7. (a)** No **(b)** Yes **8. (a)** No
(b) No **(c)** No **(d)** No **9.** $3^{-1/2}$ **10.** $7^{2/3}$ **11.** $\sqrt[3]{4^2}$
12. $\dfrac{1}{\sqrt{10^3}}$ **13.** $5^{3/5}$ **14.** $\dfrac{1}{\sqrt{8}}$ **15.** $\sqrt[5]{a^2}$ **16.** $x^{-5/2}$
17. (a) -64 **(b)** 64 **(c)** $-\frac{27}{25}$ **18. (a)** -125 **(b)** -125
(c) 4 **19. (a)** $\frac{1}{2}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{9}{4}$ **20. (a)** -8 **(b)** $-\frac{1}{8}$
(c) $-\frac{125}{27}$ **21. (a)** 625 **(b)** 25 **(c)** 64 **22. (a)** 3^{13}
(b) 1000 **(c)** 3^{20} **23. (a)** $6\sqrt[3]{2}$ **(b)** $\dfrac{\sqrt{2}}{3}$ **(c)** $\dfrac{3\sqrt{3}}{2}$
24. (a) $6\sqrt[3]{3}$ **(b)** $\dfrac{3\sqrt{2}}{5}$ **(c)** $\dfrac{2\sqrt{3}}{7}$ **25. (a)** $3\sqrt{5}$ **(b)** 4
(c) $6\sqrt[3]{2}$ **26. (a)** $8\sqrt{5}$ **(b)** 3 **(c)** $5\sqrt[3]{9}$ **27. (a)** $2\sqrt{11}$
(b) 4 **(c)** $\frac{1}{4}$ **28. (a)** $\frac{1}{2}$ **(b)** 2 **(c)** $\frac{1}{3}$ **29. (a)** x^7 **(b)** $8y^6$
(c) y^5 **30. (a)** y^7 **(b)** $64x^2$ **(c)** x **31. (a)** $\dfrac{1}{x^2}$ **(b)** $\dfrac{1}{w}$
(c) x^6 **32. (a)** $\dfrac{1}{y^3}$ **(b)** $\dfrac{1}{z^2}$ **(c)** $\dfrac{1}{y^3}$ **33. (a)** a^6 **(b)** a^{18}
(c) $\dfrac{5x^9}{8}$ **34. (a)** z^4 **(b)** $16a^{20}$ **(c)** $-54z^9$ **35. (a)** $6x^3y^5$
(b) $\dfrac{25w^4}{z}$ **36. (a)** $\dfrac{4n^2}{m^2}$ **(b)** $\dfrac{27a^{14}}{b^7}$ **37. (a)** $\dfrac{x^7}{y}$ **(b)** $\dfrac{a^9}{8b^6}$
38. (a) $\dfrac{1}{yz^3}$ **(b)** $\dfrac{y^8}{x^{12}}$ **39. (a)** $\dfrac{a^{19}b}{c^9}$ **(b)** $\dfrac{v^{10}}{u^{11}}$

40. (a) $\frac{x^{10}}{yz^4}$ (b) r^9s^2 **41.** (a) $\frac{4a^8}{b^9}$ (b) $\frac{125}{x^6y^3}$

42. (a) $5x^2y$ (b) $\frac{a^9}{8b^{12}}$ **43.** (a) $\frac{b^3}{3a}$ (b) $\frac{s^3}{q^7r^4}$

44. (a) $\frac{25t^{10}}{s^6}$ (b) $\frac{x^3y^{15}}{z^3}$ **45.** (a) $|x|$ (b) $2x^2$

46. (a) x^2 (b) xy^2 **47.** (a) $2ab\sqrt[6]{b}$ (b) $4a^2\sqrt[3]{b^2}$

48. (a) $|x|\sqrt{|yz|}$ (b) $2|x|$ **49.** (a) $7\sqrt{2}$ (b) $9\sqrt{3}$

50. (a) $8\sqrt{5}$ (b) $\sqrt[5]{2}$ **51.** (a) $(3a+1)\sqrt{a}$

(b) $(4+x^2)\sqrt{x}$ **52.** (a) $(x+2)\sqrt[3]{x}$

(b) $(20rt^2+12t)\sqrt{2rt}$ **53.** (a) $9\sqrt{x^2+1}$ (b) $6\sqrt{x^2+y^2}$

54. (a) $3\sqrt{a(3a+7)}$ (b) $5\sqrt{t(4t+3)}$

55. (a) 2 (b) -2 (c) $\frac{1}{3}$ **56.** (a) 3 (b) -2 (c) $-\frac{1}{2}$

57. (a) 4 (b) $\frac{3}{2}$ (c) $\frac{8}{27}$ **58.** (a) 25 (b) $\frac{125}{512}$ (c) $\frac{1}{81}$

59. (a) 5 (b) $\sqrt[5]{3}$ (c) 4 **60.** (a) 9 (b) $\frac{1}{7}$ (c) $\frac{1}{36}$

61. (a) x^2 (b) y^2 **62.** (a) $16b^{3/4}$ (b) $45a^2$

63. (a) $w^{5/3}$ (b) $8a^{13/4}$ **64.** (a) $\frac{1}{4y^2}$ (b) $\frac{1}{u^{4/3}v^2}$

65. (a) $4a^4b$ (b) $8a^9b^{12}$ **66.** (a) $\frac{x^3}{y^{1/5}}$ (b) r^4 **67.** (a) $4st^4$

(b) $\frac{4x}{y}$ **68.** (a) $\frac{2y^{4/3}}{x^2}$ (b) $\frac{\sqrt[4]{t}}{2\sqrt{s}}$ **69.** (a) $\frac{x^4}{y}$ (b) $\frac{8y^8}{x^2}$

70. (a) $\frac{x}{ab^{10}y^{10/3}}$ (b) $4s^{3/2}t^{9/2}$ **71.** (a) $x^{3/2}$ (b) $x^{6/5}$

72. (a) $x^{5/2}$ (b) $x^{3/2}$ **73.** (a) $y^{3/2}$ (b) $10x^{7/12}$

74. (a) $b^{5/4}$ (b) $2a^{7/6}$ **75.** (a) $2st^{11/6}$ (b) x

76. (a) xy^2 (b) $2x^{1/6}$ **77.** (a) $y^{1/2}$ (b) $\frac{4u}{v^2}$

78. (a) $s^{5/4}$ (b) $\frac{3y}{x}$ **79.** (a) $\frac{\sqrt{6}}{6}$ (b) $\frac{\sqrt{6}}{2}$

(c) $\frac{9\sqrt[4]{8}}{2}$ **80.** (a) $4\sqrt{3}$ (b) $\frac{2\sqrt{15}}{5}$ (c) $\frac{8\sqrt[3]{5}}{5}$

81. (a) $\frac{\sqrt{5x}}{5x}$ (b) $\frac{\sqrt{5x}}{5}$ (c) $\frac{\sqrt[5]{x^2}}{x}$ **82.** (a) $\frac{\sqrt{3st}}{3t}$

(b) $\frac{a\sqrt[3]{b^2}}{b}$ (c) $\frac{\sqrt[3]{c^2}}{c}$ **83.** (a) 6.93×10^7 (b) 7.2×10^{12}

(c) 2.8536×10^{-5} (d) 1.213×10^{-4} **84.** (a) 1.2954×10^8

(b) 7.259×10^9 (c) 1.4×10^{-9} (d) 7.029×10^{-4}

85. (a) 319,000 (b) 272,100,000 (c) 0.00000002670

(d) 0.000000009999 **86.** (a) 710,000,000,000,000

(b) 6,000,000,000,000 (c) 0.00855 (d) 0.0000000006257

87. (a) 5.9×10^{12} mi (b) 4×10^{-13} cm

(c) 3.3×10^{19} molecules **88.** (a) 9.3×10^7 mi

(b) 5.3×10^{-23} g (c) 5.97×10^{24} kg **89.** 1.3×10^{-20}

90. 9.14×10^{13} **91.** 1.429×10^{19} **92.** 6.3×10^{38}

93. 7.4×10^{-14} **94.** 3.19×10^{-106} **95.** (a) Negative

(b) Positive (c) Negative (d) Negative (e) Positive

(f) Negative **96.** (a) $2^{1/2}$ (b) $\left(\frac{1}{2}\right)^{1/3}$ (c) $7^{1/4}$ (d) $\sqrt{3}$

97. 2.5×10^{13} mi **98.** $8\frac{1}{3}$ min **99.** 1.3×10^{21} L

100. $52,900 **101.** 4.03×10^{27} molecules **102.** 41.3 mi

103. (a) 28 mi/h (b) 167 ft **104.** 1.5×10^{11} m

SECTION 1.3 PAGE 33

1. (a) 3; $2x^5, 6x^4, 4x^3$ (b) $2x^3$; $2x^3(x^2+3x+2)$

2. 10, 7; 2, 5; $(x+2)(x+5)$ **3.** x^2; $x^2(3x+1)$

4. $A^2+2AB+B^2$; $4x^2+12x+9$ **5.** A^2-B^2; $25-x^2$

6. $(A+B)(A-B)$; $(2x-5)(2x+5)$ **7.** $(A+B)^2$; $(x+5)^2$

8. (a) No (b) Yes (c) Yes (d) Yes **9.** Binomial; $5x^3$, 6; 3

10. Trinomial; $-2x^2, 5x, -3$; 2 **11.** Monomial; -8; 0

12. Monomial; $\frac{1}{2}x^7$; 7 **13.** Four terms; $-x^4, x^3, -x^2, x$; 4

14. Binomial; $\sqrt{2}x, -\sqrt{3}$; 1 **15.** $7x+5$ **16.** $-3-x$

17. x^2+2x-3 **18.** x^2+4x+6 **19.** $5x^3+3x^2-10x-2$

20. $7x+5$ **21.** $9x+103$ **22.** $x^2-6x+17$

23. $-t^4+t^3-t^2-10t+5$ **24.** $-3t^2+21t-22$

25. $21t^2-26t+8$ **26.** $8s^2+18s-5$ **27.** $6x^2+7x-5$

28. $14y^2-13y+3$ **29.** $2x^2+5xy-3y^2$

30. $12x^2-19xy+5y^2$ **31.** $25x^2+10x+1$

32. $49y^2-28y+4$ **33.** $4u^2+4uv+v^2$ **34.** $x^2-6xy+9y^2$

35. $4x^2+12xy+9y^2$ **36.** $r^2-4rs+4s^2$ **37.** x^2-36

38. $25-y^2$ **39.** $9x^2-16$ **40.** $4y^2-25$ **41.** $x-4$

42. $y-2$ **43.** $y^3+6y^2+12y+8$ **44.** $x^3-9x^2+27x-27$

45. $-8r^3+12r^2-6r+1$ **46.** $8y^3+36y^2+54y+27$

47. x^3+4x^2+7x+6 **48.** $2x^3+x^2+1$

49. $2x^3-7x^2+7x-5$ **50.** $2x^3-5x^2-x+1$

51. $x\sqrt{x}-x$ **52.** x^2-x **53.** y^2+y **54.** $2x-\sqrt{x}$

55. x^4-a^4 **56.** $x-y$ **57.** $a-b^2$ **58.** h^2

59. $-x^4+x^2-2x+1$ **60.** $-x^4-3x^2-4$

61. $4x^2+4xy+y^2-9$ **62.** $x^2-y^2-z^2-2yz$

63. $x(-2x^2+1)$ **64.** $x^2(3x^2-6x-1)$

65. $(y-6)(y+9)$ **66.** $(z+2)(z-3)$

67. $xy(2x-6y+3)$ **68.** $7xy^2(-x^3+2y+3y^2)$

69. $(x+7)(x+1)$ **70.** $(x+5)(x-1)$

71. $(2x-5)(4x+3)$ **72.** $(y+3)(6y-7)$

73. $(3x-1)(x-5)$ **74.** $(5x+3)(x-2)$

75. $(3x+4)(3x+8)$ **76.** $(2a+2b-1)(a+b+3)$

77. $(3a-4)(3a+4)$ **78.** $(x+1)(x+5)$

79. $(3x+y)(9x^2-3xy+y^2)$ **80.** $(a-b^2)(a^2+ab^2+b^4)$

81. $(2s-5t)(4s^2+10st+25t^2)$

82. $(1+10y)(1-10y+100y^2)$ **83.** $(x+6)^2$

84. $(4z-3)^2$ **85.** $(x+4)(x^2+1)$ **86.** $(3x-1)(x^2+2)$

87. $(x^2+1)(5x+1)$ **88.** $(9x^2+1)(2x+1)$

89. $(x+1)(x^2+1)$ **90.** $(x+1)(x^4+1)$

91. $\sqrt{x}(x-1)(x+1)$ **92.** $\frac{(3+x)(1+x)}{\sqrt{x}}$

93. $x^{-3/2}(1+x)^2$ **94.** $x(x-2)(x-1)^{3/2}$

95. $(x^2+1)^{-1/2}(x^2+3)$ **96.** $x^{-1/2}(x+1)^{-1/2}(2x+1)$

97. $6x(2x^2+3)$ **98.** $15x^3(2+x)$ **99.** $(x-4)(x+2)$

100. $(x-8)(x-6)$ **101.** $(2x+3)(x+1)$

102. $(2x-1)(x+4)$ **103.** $9(x-5)(x+1)$

104. $(4x+3)(2x+1)$ **105.** $(7-2y)(7+2y)$

106. $(2t-3s)(2t+3s)$ **107.** $(t-3)^2$ **108.** $(x+5)^2$

109. $(2x+y)^2$ **110.** $(r-3s)^2$ **111.** $4ab$ **112.** $\frac{4}{x}$

113. $(x-1)(x+1)(x-3)(x+3)$

114. $(a-1)(a+1)(b-2)(b+2)$

115. $(2x-5)(4x^2+10x+25)$

116. $(x^2+4)(x^4-4x^2+16)$ **117.** $x(x+1)^2$

118. $3x(x-3)(x+3)$ **119.** $x^2y^3(x+y)(x-y)$

120. $2xy^3(9x-y)$ **121.** $(x-2)(x+2)(3x-1)$

122. $(3x-1)(3x+1)(x+2)$ **123.** $3(x-1)(x+2)$

124. $y^4(y+2)^3(y+1)^2$ **125.** $(a-1)(a+1)(a-2)(a+2)$

126. $(a-1)(a+3)(a+1)^2$