Name ~~Key~~ Teacher ________ Hour ________

UNIT Final Exam

Final Exam Review Algebra 2C

Module 10, 11, 13, 15/16, Trig. and 17 Exam Review

MODULE 10 REVIEW

Find the inverse function and state the domain and range of the inverse function. Then graph the inverse function given $f(x)$.

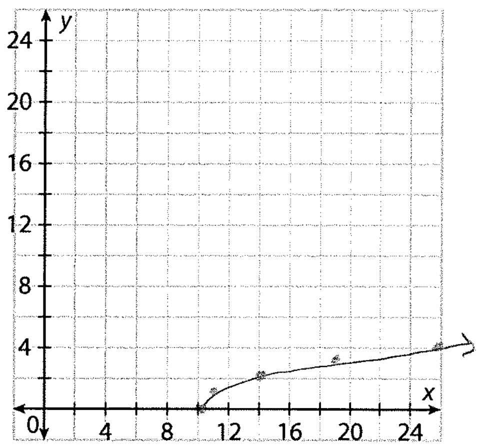
1. $f(x) = x^2 + 10$

$x = y^2 + 10$

$x - 10 = y^2$

$f^{-1}(x) = \sqrt{x-10}$ D: $x \geq 10$

R: $y \geq 0$



2. Find the inverse function and state the domain and range of the inverse function.

$f(x) = x^2 - \frac{3}{4}$

$x = y^2 - \frac{3}{4}$

$x + \frac{3}{4} = y^2$

$f^{-1}(x) = \sqrt{x + \frac{3}{4}}$

D: $x \geq -\frac{3}{4}$

R: $y \geq 0$

3. Find the inverse function and state the domain and range of the inverse function.

$f(x) = x^3 - \frac{5}{6}$

$x = y^3 - \frac{5}{6}$

$x + \frac{5}{6} = y^3$

$f^{-1}(x) = \sqrt[3]{x + \frac{5}{6}}$

D: $\mathbb{R}$

R: $\mathbb{R}$

4. Find the inverse function and state the domain and range of the inverse function.

$f(x) = x^3 + 9$

$x = y^3 + 9$

$x - 9 = y^3$

$f^{-1}(x) = \sqrt[3]{x-9}$

D: $\mathbb{R}$

R: $\mathbb{R}$

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For questions 5 – 8 describe the transformations of $g(x)$ from the parent function $f(x) = \sqrt{x}$.

5. $g(x) = \sqrt{\frac{1}{2}x} + 1$

Up 1
Horz. stretch by 2

6. $g(x) = -5\sqrt{x+1} - 3$

reflection over x-axis
Vert. stretch by 5
Left 1
Down 3

7. $g(x) = \frac{1}{4}\sqrt{x-5} - 2$

Vert. comp. by $\frac{1}{4}$
Right 5
Down 2

8. $g(x) = \sqrt{-7(x-7)}$

reflection over the y-axis
Horz. comp. by $\frac{1}{7}$
Right 7

For questions 9 and 10 describe the domain and range of each function using set notation.

9. $g(x) = 3\sqrt{x+4} + 3$

D: $x \geq -4$ or $\{x \mid x \geq -4\}$
R: $y \geq 3$ or $\{y \mid y \geq 3\}$

10. $g(x) = -7\sqrt{x-3} - 5$

D: $x \geq 3$ or $\{x \mid x \geq 3\}$
R: $y \geq -5$ or $\{y \mid y \geq -5\}$

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For questions 11 and 12 plot the transformed function $g(x)$ on the grid with the parent function, $f(x) = \sqrt{x}$. Describe the domain and range of each function using set notation.

11. $g(x) = -\sqrt{x} + 3$

a = -1

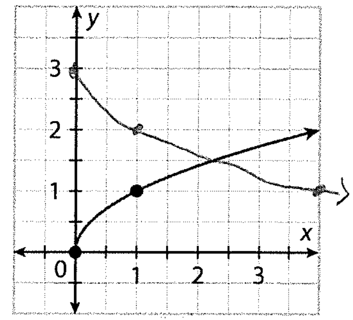
h = 0 K = 3

Starting point (0, 3)

D: x ≥ 0

R: y ≤ 3

	h	h+1	h+4
x	0	1	4
y	3	2	1
	K	K+a	K+2a



12. $g(x) = 4\sqrt{x+3} - 4$

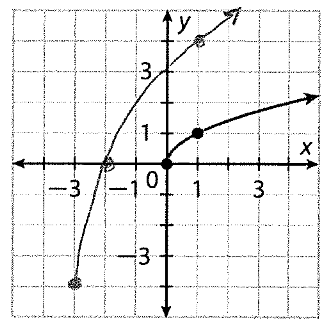
a = 4 h = -3 K = -4

Starting point (-3, -4)

D: x ≥ -3

R: y ≥ -4

		h+1	h+4
x	-3	-2	1
y	-4	0	4
	K	K+a	K+2a

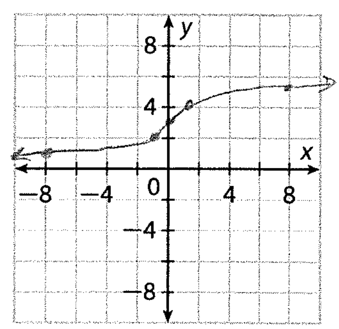


13. Graph the function $g(x) = \sqrt[3]{x} + 3$. Identify the domain and range of the function.

Up 3

D: ℝ R: ℝ

	h-8	h-1	h	h+1	h+8
x	-8	-1	0	1	8
y	1	2	3	4	5
	K-2	K-1	K	K+1	K+2



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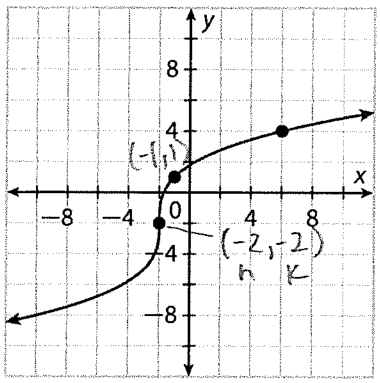
For questions 14 and 15 use the given graphs, write a cube root function. Write the function in the form $g(x) = a\sqrt[3]{x-h}+k$.

14.



$h = -2$

$K = -2$

$x = -1$

$y = 1$

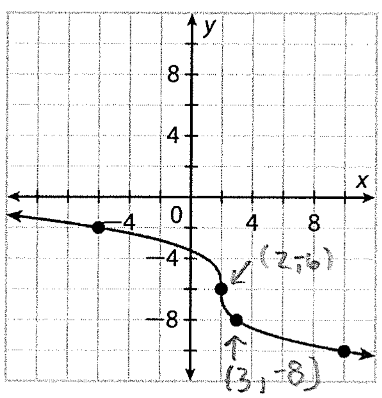
$1 = a\sqrt[3]{-1+2} - 2$

$3 = a\sqrt[3]{1}$

$3 = a$

$g(x) = 3\sqrt[3]{x+2} - 2$

15.



$h = 2$

$k = -6$

$x = 3$

$y = -8$

$-8 = a\sqrt[3]{3-2} - 6$

$+6 \qquad +6$

$-2 = a\sqrt[3]{1}$

$-2 = a$

$g(x) = -2\sqrt[3]{x-2} - 6$

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MODULE 11 REVIEW

Translate expressions with rational exponents into radical expressions. Simplify numerical expressions when possible. Assume all variables are positive.

16. $64^{\frac{5}{3}}$ $\left(\sqrt[3]{64}\right)^5 = (4)^5 = 1024$

17. $3^{\frac{2}{7}}$ $\sqrt[7]{3^2} = \sqrt[7]{9}$

18. $vw^{\frac{2}{3}}$ $v \cdot w^{\frac{2}{3}} = v\sqrt[3]{w^2}$

Translate radical expressions into expressions with rational exponents. Simplify numerical expressions when possible. Assume all variables are positive.

19. $\sqrt[7]{y^5}$ $y^{\frac{5}{7}}$

20. $\sqrt[5]{32^2} = (32)^{\frac{2}{5}} = 4$

21. $\sqrt[3]{\left(\frac{4}{x}\right)^9} = \left(\frac{4}{x}\right)^{\frac{9}{3}} = \left(\frac{4}{x}\right)^3 = \frac{(4)^3}{(x)^3} = \frac{64}{x^3}$

Simplify the expression. Assume that all variables are positive.

22. $\left(\frac{16^{\frac{5}{3}}}{16^{\frac{5}{6}}}\right)^{\frac{9}{5}} = \left(16^{\frac{10}{6}-\frac{5}{6}}\right)^{\frac{9}{5}} = \left(16^{\frac{5}{6}}\right)^{\frac{9}{5}} = (16)^{\frac{9}{6}} = (16)^{\frac{3}{2}} = 64$

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Simplify the expression. Assume that all variables are positive.

23. $\frac{9^{\frac{3}{2}} \cdot 9^{\frac{1}{2}}}{9^{-2}}$ $= \frac{9^{\frac{3}{2}+\frac{1}{2}}}{9^{-2}} = \frac{9^{2}}{9^{-2}} = 9^{2-(-2)} = 9^{4} = 6561$

24. $\frac{2xy}{\left(x^{\frac{1}{3}}y^{\frac{2}{3}}\right)^{\frac{3}{2}}}$ $= \frac{2x^{1}y^{1}}{x^{\frac{5}{6}}y^{1}} = \frac{2x^{1}y^{1}}{x^{\frac{1}{2}}y^{1}} = 2x^{1-\frac{1}{2}}y^{1-1} = 2x^{\frac{1}{2}}$

Solve the equations

25. $\left((x+4)^{\frac{1}{2}}\right)^{2} = (6)^{2}$

$x + 4 = 36$

$x = 32$

26. $\left((x-6)^{\frac{1}{2}}\right)^{2} = (x-2)^{2}$

$x - 6 = x^2 - 4x + 4$

$-x + 6 \quad -x + 6$

$0 = x^2 - 5x + 10$

$x = \frac{5 \pm \sqrt{-15}}{2}$

$(25) - 4(1)(10) = -15$

No solution

27. $5 - \sqrt[3]{x-4} = 2$

$-5 \quad -5$

$-\sqrt[3]{x-4} = -3$

$\sqrt[3]{x-4} = 3$

$x - 4 = 27$

$\boxed{x = 31}$

28. $\left((5x+1)^{\frac{1}{4}}\right)^{4} = (4)^{4}$

$5x + 1 = 256$

$5x = 255$

$x = 51$

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MODULE 13 REVIEW

State the domain and range of the given function. Then identify the new values of the reference points and the asymptote. Use these values to graph the function.

29. $h(x) = 2(3^{x+2}) - 1$

$a=2 \quad b=3 \quad h=-2 \quad k=-1$

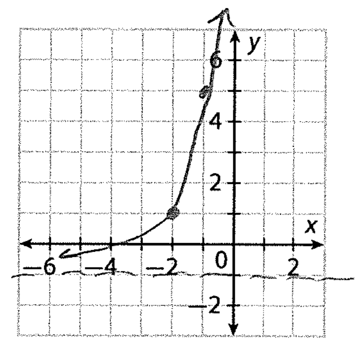
$(h, a+k) = (-2, 1)$

$(h+1, ab+k) = (-1, 5)$

D: $\mathbb{R}$

R: $y > -1$

Asy. @ $y = -1$



30. $f(x) = 3(6^{x-7}) - 8$

$a=3 \quad b=6 \quad h=7 \quad k=-8$

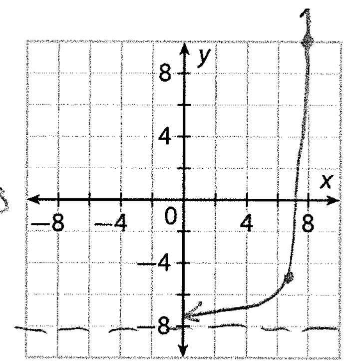
$(h, a+k) = (7, -5)$

$(h+1, ab+k) = (8, 10)$

D: $\mathbb{R}$

R: $y > -8$

Asy @ $y = -8$



Describe the transformation(s) from each parent function and give the domain and range of each function.

31. $g(x) = -\left(\frac{1}{10}\right)^{x-1} + 2$

reflection over x-axis
Right 1
Up 2

D: $\mathbb{R}$

R: $y < 2$

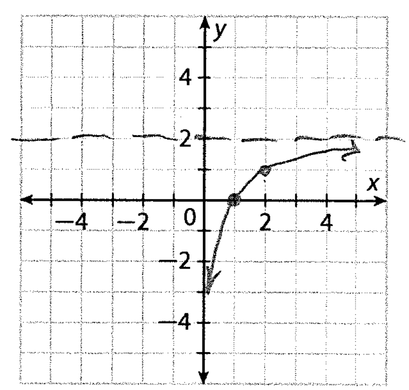
Graph the given transformed function. Then describe the domain and range of the transformed function using set notation.

32. $g(x) = -2\left(\frac{1}{2}\right)^{x-1} + 2$

$a=-2 \quad b=\frac{1}{2} \quad h=1 \quad k=2$

$(h, a+k) = (1, 0)$

$(h+1, ab+k) = (2, 1)$



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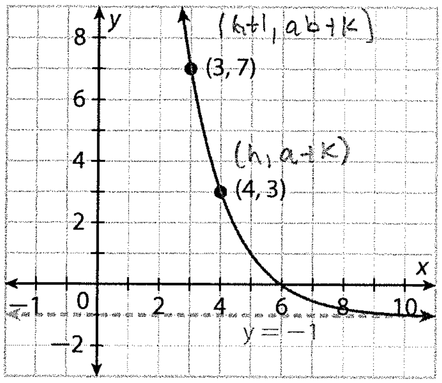
Write the function represented by each graph and state the domain and range using set notation.

33.



$h = 4$
$k = -1$

$y = a(b)^{x-h} + k$

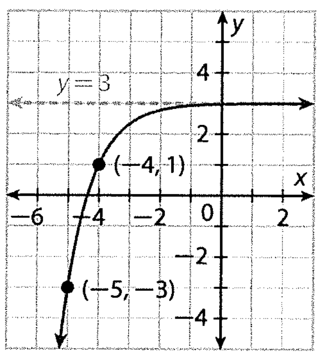
$3 = a - 1$
$a = 4$

$7 = 4b - 1$
$8 = 4b$
$b = 2$

$g(x) = 4(2)^{x-4} - 1$

D: $\mathbb{R}$ R: $y > -1$

34.



$h = -4$
$k = 3$

$1 = a + 3$
$a = -2$

$-3 = -2(b) + 3$
$-6 = -2(b)$
$b = 3$

$g(x) = -2(3)^{x+4} + 3$

D: $\mathbb{R}$
R: $y < 3$

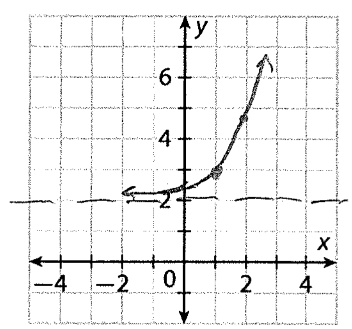
Given the function of the form $g(x) = a \cdot e^{x-h} + k$, identify the reference points and use them to draw the graph. State the domain and range in set notation.

35. $g(x) = e^{x-1} + 2$

$a = 1$ $b = e$ $h = 1$ $k = 2$

$(h, a+k) = (1, 3)$
$(h+1, ae+k) = (2, e+2)$

D: $\mathbb{R}$ R: $y > 2$



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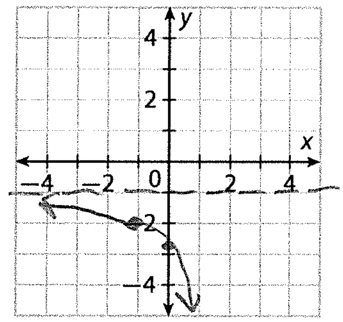
Given the function of the form $g(x) = a \cdot e^{x-h} + k$, identify the reference points and use them to draw the graph. State the domain and range in set notation.

36. $g(x) = -e^{x+1} - 1$

$a = -1 \quad b = e \quad h = -1 \quad K = -1$

$(h, a+k) = (-1, -2)$

$(h+1, ae+k) = (0, -e-1)$



D: $\mathbb{R}$ R: $y < -1$

37. A person invests \$2560 in an account that earns 5.2% annual interest. Find when the value of the investment reaches \$6000.

$A = 2560(1 + .052)^t$

$A = 6000$

$t = 16.8$ years

38. A person invests \$200 in an account that earns 1.98% annual interest compounded quarterly. Find when the value of the investment reaches \$500.

$A = 200\left(1 + \frac{.0198}{4}\right)^{4t}$

$A = 500$

$t = 49.4$ years

MODULE 15 REVIEW

39. Consider the exponential function $f(x) = 3^x$.

a. State the function's domain and range using set notation.

D: $\mathbb{R}$ R: $y > 0$

b. Describe any restriction you must place on the domain of the function so that its inverse is also a function.

$x > 0$

c. Write the rule for the inverse function.

$f^{-1}(x) = \log_3 x$

d. State the inverse function's domain and range using set notation.

D: $x > 0$ R: $\mathbb{R}$

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40. Consider the logarithmic function $f(x) = log_4 x$.

a. State the function's domain and range using set notation.

D: $x > 0$ R: $\mathbb{R}$

b. Describe any restriction you must place on the domain of the function so that its inverse is also a function.

None

c. Write the rule for the inverse function.

$f^{-1}(x) = 4^x$

d. State the inverse function's domain and range using set notation.

D: $\mathbb{R}$ R: $y > 0$

Write the given exponential equation in logarithmic form.

41. $3^m = n$

$\log_3 n = m$

42. $\left(\frac{1}{2}\right)^p = q$

$\log_{\frac{1}{2}} q = p$

Write the given logarithmic equation in exponential form.

43. $\log_8 x = y$

$8^y = x$

44. $\log_{\frac{2}{3}} c = d$

$\left(\frac{2}{3}\right)^d = c$

Evaluate the following Logarithms for the given values

45. If $f(x) = \log_3 x$, find $f(243)$, $f\left(\frac{1}{27}\right)$, and $f(\sqrt{27})$.

$\log_3(243) = 5$ $\log_3\left(\frac{1}{27}\right) = -3$ $\log_3(\sqrt{27}) = \frac{3}{2}$

46. If $f(x) = \log_{\frac{1}{4}} x$, find $f\left(\frac{1}{64}\right)$, $f(256)$, and $f(\sqrt[3]{16})$

$\log_{\frac{1}{4}}\left(\frac{1}{64}\right) = 3$ $\log_{\frac{1}{4}}(256) = -4$ $\log_{\frac{1}{4}}(\sqrt[3]{16}) = -\frac{2}{3}$

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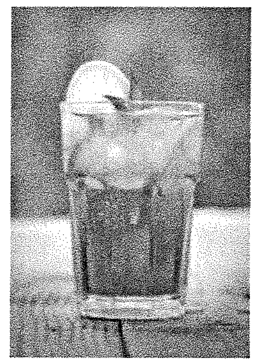
Use a calculator to find the common logarithm and the natural logarithm of the given number.

47. 19 $\log(19) = 1.28$ $\ln(19) = 2.94$

48. 9 $\log(9) = .95$ $\ln(9) = 2.2$

49. The acidity level, or pH, of a liquid is given by the formula $\text{pH} = \log \frac{1}{[\text{H}^+]}$ where $[\text{H}^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. What is the pH of iced tea with a hydrogen ion concentration of 0.000158 mole per liter?

$pH = \log\left(\frac{1}{.000158}\right) = 3.8$



50. Identify the transformations of the graph of $f(x) = \log_b x$ that produce the graph of the given function g(x). Then graph g(x) on the same coordinate plane as the graph of $f(x)$ by applying the transformations to the asymptote x = 0 and to the reference points (1, 0) and (b, 1). Also state the domain and range of $g(x)$ using set notation.

$g(x) = -4\log_2(x+2) + 1$ $a=-4$ $b=2$ $h=-2$ $k=1$

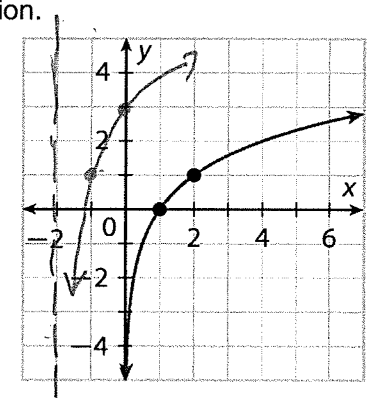
reflection over x-axis
Vert. Stretch of 4
Left 2
Up 1

$(1+h, k) = (-1, 1)$

D: $x > -2$

$(b+h, a+k) = (0, -3)$

R: $\mathbb{R}$



51. $g(x) = 3\log(x-1) - 1$ $a=3$ $h=1$ $k=-1$ $b=10$

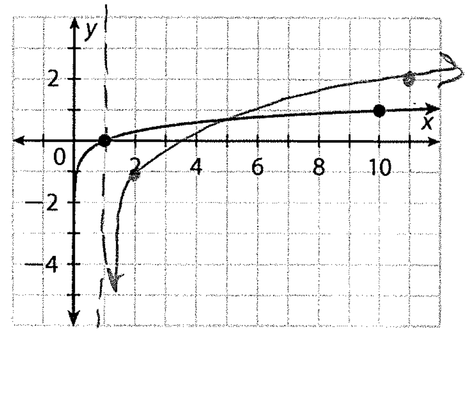
Vert. stretch by 3
Right 1
Down 1

$(1+h, k) = (2, -1)$

D: $x > 1$

$(b+h, a+k) = (11, 2)$

R: $\mathbb{R}$



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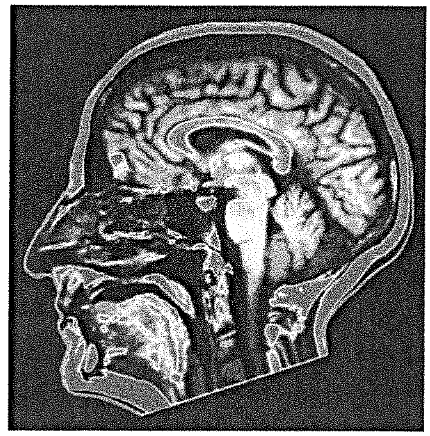
SKIP

~~52.~~ The radioactive isotope fluorine-18 is used in medicine to produce images of internal organs and detect cancer. It decays to the stable element oxygen-18. The table gives the percent of fluorine-18 that remains in a sample over a period of time.

Time (hours)	Percent of Fluorine-18 Remaining
0	100
1	68.5
2	46.9
3	32.1



a. Write an exponential model for the percent of fluorine-18 remaining as a function of time (in hours).

b. Find the inverse of the exponential model after rewriting it with a base of e. Describe what information the inverse gives.

c. Perform logarithmic regression on the data (using the percent of fluorine-18 remaining as the independent variable and time as the dependent variable). Compare this model with the inverse model from part b.

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MODULE 16 REVIEW

Express each expression as a single logarithm. Simplify if possible.

53. $\log_9 12 + \log_9 546.75$ $= \log_9(6561) = 4$

$\log_9(12 \cdot 546.75)$

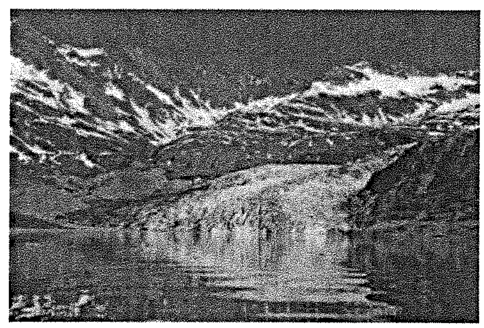
54. $\log_2 76.8 - \log_2 1.2$

$\log_2\left(\frac{76.8}{1.2}\right) = \log_2(64) = 6$

55. $\log_{11} 11^{23}$ $23\log_{11}(11) = 23(1) = 23$

56. Solve the following problem using logarithmic models

Geology Seismologists use the Richter scale to express the energy, or magnitude, of an earthquake. The Richter magnitude of an earthquake M is related to the energy released in ergs E shown by the formula $M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$. In 1964, an earthquake centered at Prince William Sound, Alaska registered a magnitude of 9.2 on the Richter scale. Find the energy released by the earthquake.



$M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$

$\frac{2M}{3}\ \log_{10}\left(\frac{E}{10^{11.8}}\right)$

$10^{\left(\frac{2M}{3}\right)} = \frac{E}{10^{11.8}}$

$E = 10^{\left(\frac{2M}{3}\right)} \cdot 10^{11.8}$

$E = 10^{\frac{2M}{3} + 11.8}$

$E = 10^{\frac{2(9.2)}{3} + 11.8}$

$E = 10^{17.93}$ ergs

57. The population P of bacteria in a culture after t minutes is given by the equation $P = P_0(1.12)^t$, where P_0 is the initial population. If the number of bacteria starts at 200, how long will it take for the population to increase to 1000?

$\frac{1000}{200} = \frac{200(1.12)^t}{200}$

$5 = (1.12)^t$

$\log(5) = t\log(1.12)$

$t = \frac{\log(5)}{\log(1.12)} = 14.2$

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Solve the equations. Give the exact solution and an approximate solution to three decimal places.

58. $6^{3x-9} - 10 = -3$

$6^{3x-9} = 7$

$(3x-9)\log(6) = \log(7)$

$3x - 9 = \dfrac{\log(7)}{\log(6)}$

$3x = \left(\dfrac{\log(7)}{\log(6)}\right) + 9$

$x = \dfrac{\left(\dfrac{\log(7)}{\log(6)}\right) + 9}{3}$

$x = 3.362$

59. $7e^{3x} = 42$

$e^{3x} = 6$

$3x = \ln(6)$

$x = \dfrac{\ln(6)}{3}$

$x = .597$

60. $11^{6x+2} = 12$

$(6x+2)\log(11) = \log(12)$

$6x + 2 = \left(\dfrac{\log(12)}{\log(11)}\right)$

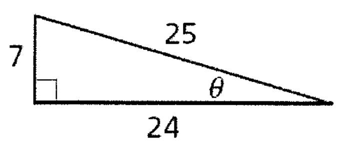
$6x = \left(\dfrac{\log(12)}{\log(11)}\right) - 2$

$x = \dfrac{\left(\dfrac{\log(12)}{\log(11)}\right) - 2}{6}$

$x = -.161$

TRIG MODULE REVIEW

61. Find the sinθ, cosθ, and tanθ. Make sure the ratio is a reduced fraction.



$\sin\theta = \dfrac{7}{25}$ $\cos\theta = \dfrac{24}{25}$ $\tan\theta = \dfrac{7}{24}$

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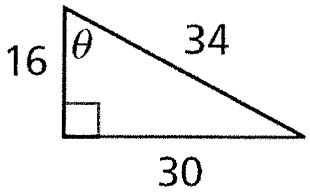
62. Find the sinθ, cosθ, and tanθ. Make sure the ratio is a reduced fraction.

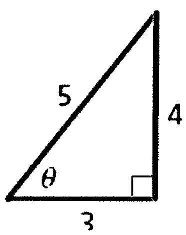


$\sin\theta = \frac{30}{34} = \frac{15}{17}$

$\cos\theta = \frac{16}{34} = \frac{8}{17}$

$\tan\theta = \frac{30}{16} = \frac{15}{8}$

63. Find the cscθ, secθ, and cotθ. Make sure the ratio is a reduced fraction.

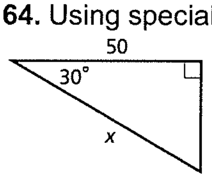


$\sin\theta = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\cot\theta = \frac{3}{4}$

64. Using special right triangles find the length of x. Leave your answer as a simplified radical.

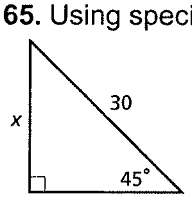


$\cos 30 = \frac{50}{x}$ $x\sqrt{3} = 100$

$\frac{\sqrt{3}}{2} = \frac{50}{x}$ $x = \frac{100}{\sqrt{3}}$

$x = \frac{100\sqrt{3}}{3}$

65. Using special right triangles find the length of x. Leave your answer as a simplified radical.



$\sin(45) = \frac{x}{30}$ $x = \frac{30\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2} = \frac{x}{30}$ $x = 15\sqrt{2}$

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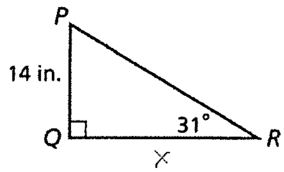
66. Find the length of the given side. Round your answer to the nearest tenth.

QR



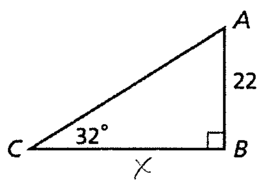
$\tan(31) = \frac{14}{x}$

$x = \frac{14}{\tan(31)}$

$x = 23.3$

67. Find the length of the given side. Round your answer to the nearest tenth.

BC

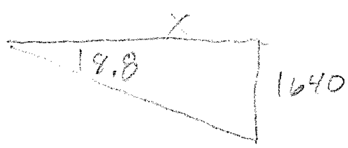


$\tan(32) = \frac{22}{x}$

$x = \frac{22}{\tan(32)}$

$x = 35.2$

68. The Pilot of a helicopter measures the angle of depression to a landing spot to be 18.8°. If the pilots altitude is 1640 meters, what is the horizontal distance to the landing spot. Round your answer to the nearest meter.

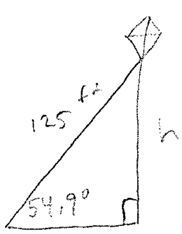


$\tan(18.8) = \frac{1640}{x}$

$x = \frac{1640}{\tan(18.8)}$

$x = 4817$ meters

69. A kite string is 125 feet long. The angle between the kite string and the ground is 54.9°. How high is the kite?



$\tan(54.9) = \frac{h}{125}$

$h = 125 \cdot \tan(54.9)$

$h = 177.9$ ft or 178 ft

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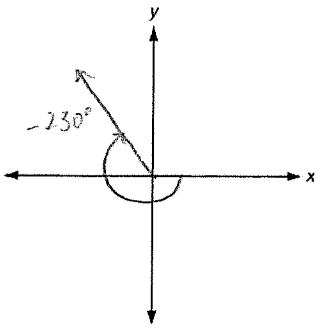
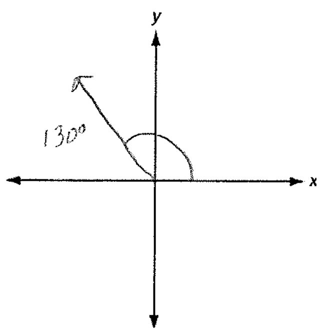
70. Draw an angle with the given measure in standard position.
$-230°$



71. Draw an angle with the given measure in standard position.
$130°$



72. Find the measures of a positive angle and a negative angle that are coterminal with each given angle.
$\theta = 390°$

$\theta = 390 - 360 = 30°$

$\theta = 390 - 720 = -330°$

73. Find the measures of a positive angle and a negative angle that are coterminal with each given angle.
$\theta = -150°$

$\theta = -150 + 360 = 210°$

$\theta = -150 - 360 = -510°$

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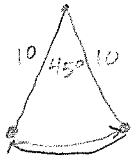
74. A pendulum is 10 feet long. Its central angle is 45°. The pendulum makes one back and forth swing every 15 seconds. To the nearest foot, how far does the pendulum swing each minute?

8 trips in 1 min



$S = r\theta = 1$ trip

$= 10\left(\frac{\pi}{4}\right)$

$= 7,953$

$d = 8 \cdot s$

$= 8 \cdot (7.583)$

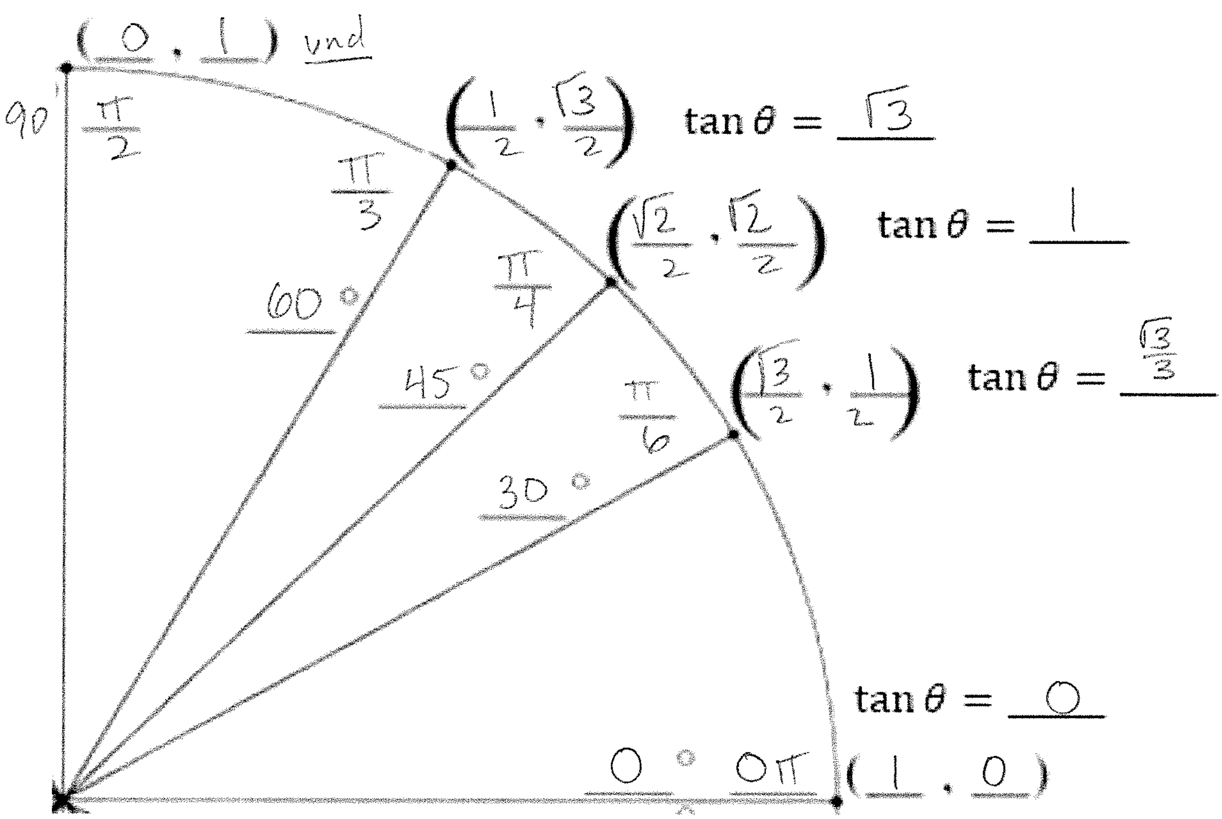
$d = 62.83$ ft

75. Grand Ledge, Michigan is located about 43° north of the equator. If Earth's radius is about 3959 miles, approximately how many miles is Grand Ledge from the equator? Hint use the arc length formula to solve $s = r\theta$. Round your answer to the nearest mile.

$\theta = \frac{43 \cdot \pi}{180}$ $r = 3959$

$S = (3959) \cdot \left(\frac{43\pi}{180}\right) = 2971$ mi

76. Fill in the first quadrant of the unit circle



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For questions 77 – 80. Find the measure of the reference angle for each given angle and give your answer in degrees only.

77. $\theta = -72°$ ref∠ = 72°

78. $\theta = 225°$ ref∠ = 45°

79. $\theta = -\frac{7\pi}{6}$ ref∠ = 30°

80. $\theta = \frac{3\pi}{5}$ ref∠ = 72°

For questions 81 – 86 without using a calculator find the exact value of each trigonometric function. Show your work!

81. $\cos 120°$

Q2

ref∠ = 60

$-\cos(60) = -\frac{1}{2}$

82. $\sin -135°$

Q3

ref∠ = 45

$-\sin(45) = -\frac{\sqrt{2}}{2}$

83. $\tan 405°$

Q1 ref∠ = 45

$\tan(45) = 1$

84. $\sin\frac{2\pi}{3}$

Q2

ref∠ = 60

$\sin(60) = \frac{\sqrt{3}}{2}$

85. $cos\frac{5\pi}{2}$

pos. y-axis

ref∠ = 90

$\cos(90) = 0$

86. $tan\frac{-11\pi}{6}$

Q1

ref∠ = 30

$\tan(30) = \frac{\sqrt{3}}{3}$