Name Key

ALGEBRA 2B – Final Exam Review
Modules 5 – 9, and 12

Describe the transformations of how the graph of g(x) is related to the graph of $f(x) = x^3$.

1. $g(x) = -3(x-4)^3 + 1$

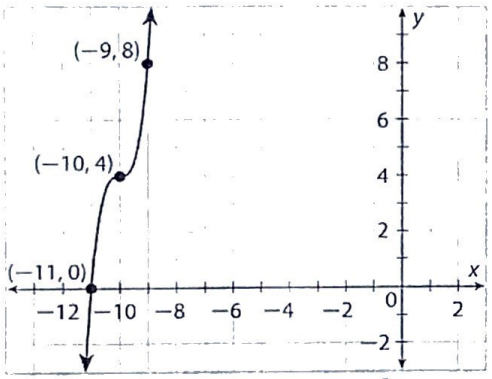
- reflection over x-axis
- vertical stretch of 3
- right 4
- up 1

2. $g(x) = \frac{1}{2}(x-2)^3 - 4$

- vertical comp. $\frac{1}{2}$
- right 2
- down 4

Write the equation of the cubic function whose graph is shown.

3.



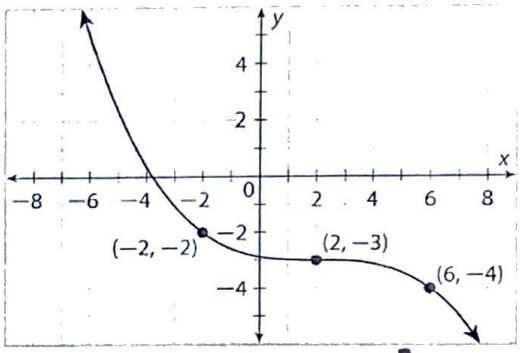
$g(x) = a(x+10)^3 + 4$

$8 = a(-9+10)^3 + 4$

$4 = a$

$g(x) = 4(x+10)^3 + 4$

4.



$g(x) = a(x-2)^3 - 3$

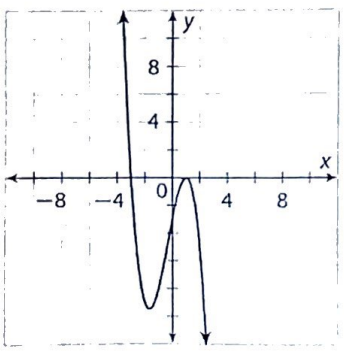
$-4 = a(6-2)^3 - 3$

$-1 = a\,64$

$a = \frac{-1}{64}$

$g(x) = \frac{-1}{64}(x-2)^3 - 3$

5. Identify the x-intercepts, domain, range and end behavior of the following graph



x-intercept(s): $-3, 1$

domain: $-\infty < x < +\infty$

range: $-\infty < y < +\infty$

end behavior: As $x \to -\infty$, $f(x) \to +\infty$

As $x \to +\infty$, $f(x) \to -\infty$

Describe the end behavior.

6. $f(x) = -2x^2 + 3x - 10$

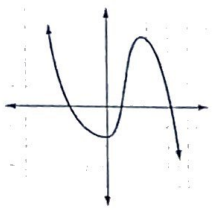
As $x \to +\infty, f(x) \to$ $-\infty$
As $x \to -\infty, f(x) \to$ $-\infty$

7. $g(x) = 5x^3 - 4x + 7$

As $x \to +\infty, f(x) \to$ $+\infty$
As $x \to -\infty, f(x) \to$ $-\infty$

Identify whether the function has an **odd or even degree** and a **positive or negative leading coefficient.**

8.



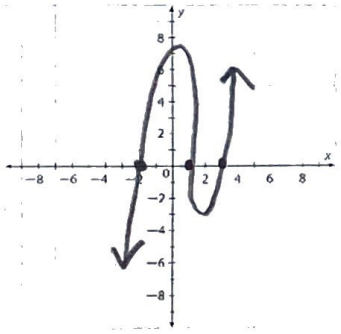
odd
negative

9. $f(x) = -x^{10}$

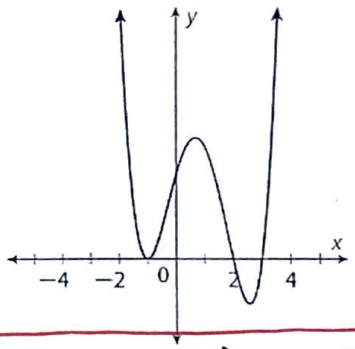
even
negative

10. Sketch a graph of the following function:

$f(x) = (x+2)(x-3)(x-1)$

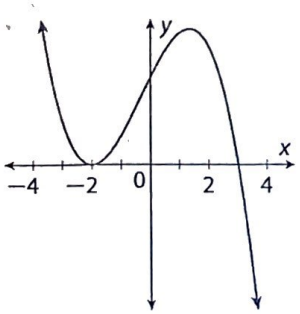


11. Write a quartic function in interval form of the following graph:



$f(x) = (x+1)^2(x-2)(x-3)$

12. Use the graph to identify the zeros and write in intercept form.



Zeros: $-2, 3$

Intercept form: $-(x+2)^2(x-3)$

Add or subtract the following polynomials.

13. $(5x^4 + x^2) + (7 + 9x^2 - 2x^4 + x^3)$

$3x^4 + x^3 + 10x^2 + 7$

14. $(x^2 - 8) + (3x^3 + 6x + 4 + 9x^2)$

$-3x^3 - 8x^2 + 6x - 4$

15. $(x + 1 - 3x^2) + (8x + 21x^2 - 1)$

$18x^2 + 9x$

16. $(9x - 12x^3) + (5x^3 + 7x + 2)$

$-17x^3 + 2x + 2$

Multiply the following polynomials.

17. $(x+2)(y^2 + 2y - 12)$

$xy^2 + 2xy - 12x + 2y^2 + 4y - 24$

18. $(5x + 3)(5x - 3)$

$25x^2 - 9$

19. $(4x + 7y)^2$

$16x^2 + 56xy + 49y^2$

20. $(2x^2 + xy - y)(y^2 + 3x)$

$2x^2y^2 + 6x^3 + xy^3 + 3x^2y - y^3 - 3xy$

Expand the product using the Binomial theorem.

21. $(x - 3)^4$

1 4 6 4 1

x^4 x^3 x^2 x 1

1 (-3) $(-3)^2$ $(-3)^3$ $(-3)^4$

9 −27 81

$x^4 - 12x^3 + 54x^2 - 108x + 81$

22. $(2x + 4)^3$

1 3 3 1

$(2x)^3$ $(2x)^2$ $2x$ 1

$8x^3$ $4x^2$

1 4 4^2 4^3

16 64

$8x^3 + 48x^2 + 96x + 64$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

23. A rectangular shape pool has a length of (3a +16) feet and a width of 5a feet. Find the area and the perimeter of the pool. Sketch a picture to help you.

$5a(3a+16)$

Area = $15a^2 + 80a$

Perimeter = $16a + 32$ $\quad 3a+16+3a+16+5a+5a$

Factor out the GCF.

24. $3b^7 + 12b^4 + 12b$

$3b(b^6 + 4b^3 + 4)$

25. $50v^6 + 60v^3 + 18$

$2(25v^6 + 30v^3 + 9)$

Factor each polynomial by grouping.

26. $(x^3 + 8x^2 + 6x + 48)$

$x^2(x+8) + 6(x+8)$

$(x^2+6)(x+8)$

27. $(4x^4 - 4x^3 - x + 1)$

$4x^3(x-1) - 1(x-1)$

$(x-1)(4x^3-1)$

Completely factor the following polynomials.

28. $x^2 + 3x - 18$

$(x+6)(x-3)$

29. $h^4 - 16$

$(h^2+4)(h^2-4)$

$(h^2+4)(h+2)(h-2)$

30. $27x^3 + 64$

$(3x+4)(9x^2 - 12x + 16)$

31. $3x^2 - 13x - 10$

$(3x^2 - 15x + 2x - 10)$

$3x(x-5) + 2(x-5)$

$(3x+2)(x-5)$

-30

$-15 \quad 2$

32. $4r^2 + 24r + 36$

$(2r+6)^2$

33. $4x^4 + 4x^3 - 24x^2$

$4x^2(x^2 + x - 6)$

$4x^2(x+3)(x-2)$

Divide using long division or synthetic division.

34. $(3x^3+9x^2-14)\div(x+3)$

$-3 \mid 3 \;\; 9 \;\; 0 \;\; -14$

$\quad\quad -9 \;\; 0 \;\; 0$

$3 \;\; 0 \;\; 0 \mid -14$

$(3x^2)(x+3)-14$

35. $(x^2-x-6)\div(x-3)$

$3 \mid 1 \;\; -1 \;\; -6$

$\quad\quad 3 \;\; 6$

$1 \;\; 2 \mid 0$

$(x+2)(x-3)$

Use synthetic substitution to evaluate P(x) for the given value.

36. $P(x)=-3x^4+5x^3-x+7$ for $x=-2$

$-2 \mid -3 \;\; 5 \;\; 0 \;\; -1 \;\; 7$

$\quad\quad 6 \;\; -22 \;\; 44 \;\; -86$

$-3 \;\; 11 \;\; -22 \;\; 43 \mid -79$

Solve the polynomial equation by finding all the rational roots.

37. $(x^3-x^2)(-x+1)=0$

$x^2(x-1)-1(x-1)=0$

$(x-1)(x^2-1)=0$

$(x-1)(x+1)(x-1)=0$

$x=1 \quad x=-1$

38. $x^4+3x^3-4x^2=0$

$x^2(x^2+3x-4)=0$

$x^2(x+4)(x-1)=0$

$x=0 \quad x=-4 \quad x=1$

39. Find the domain, range, vertical asymptote, and horizontal asymptote of the following function:

$$f(x)=\frac{1}{x-5}+8$$

Vertical Asymptote: $x=5$

Horizontal Asymptote: $y=8$

Domain: $\{x \mid x \neq 5\}$

Range: $\{y \mid y \neq 8\}$

40. A group of friends would like to go to Michigan's Adventure but must rent a car to go. The car rental is $186 and the park entrance fee is $45. If 6 people go on the trip, how much does each person pay?

$$\frac{186}{6}+45=\$76$$

List the vertical asymptote(s) and point(s) of discontinuity (if any) of the functions.

41. $f(x) = \frac{(x-5)(x+9)}{(x+5)(x-1)}$

Vertical Asymptote(s): $x = -5, x = 1$

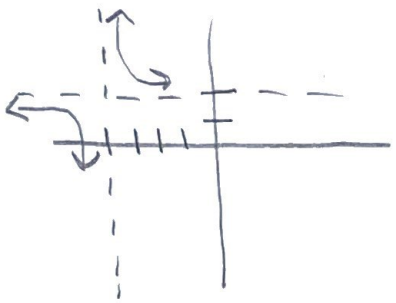
Point(s) of Discontinuity: none

42. $f(x) = \frac{(x-6)(x+4)(x+3)}{(x+4)(x-4)(x-6)}$

Vertical Asymptote(s): $x = 4$

Point(s) of Discontinuity: $x = 6, x = -4$

43. Describe the end behavior of the function $f(x) = \frac{2x-5}{x+4}$.



As $x \to -\infty$, $f(x) \to 2^-$

As $x \to +\infty$, $f(x) \to 2^+$

As $x \to -4^-$, $f(x) \to -\infty$

As $x \to -4^+$, $f(x) \to +\infty$

44. Emily and Nicole spent 4 hours canoeing on a stream. They went 3 miles upstream and then 3 miles back downstream. In still water, the average speed of the canoe was 5 miles per hour. The time T spent canoeing can be modeled by the function $T(s) = \frac{3}{5-s} + \frac{3}{5+s} = \frac{30}{(5-s)(5+s)}$, where s is the average speed of the current. A sketch of the graph is shown below. What was the average speed of the current during their trip?

≈ 4.2 mph

Find the excluded values.

45. $\frac{3x^2-1}{x^2-3x-18}$

$(x-6)(x+3)$

$x \neq 6$
$x \neq -3$

46. $\frac{x+4}{x^2-x-12}$

$(x-4)(x+3)$

$x \neq 4$
$x \neq -3$

Add or subtract.

47. $\frac{3x}{(x-5)(x+6)} - \frac{4}{(x-5)(x+6)}$

$\frac{3x^2-15x-4}{(x-5)(x+6)}$

48. $\frac{3}{x+2} + \frac{(x+1)(x+2)}{(x-5)(x+2)}$

$3x-15+x^2+3x+2$

$\frac{x^2+6x-13}{(x-5)(x+2)}$

Multiply or Divide.

49. $\frac{x+2}{(x-4)(x+5)} \cdot \frac{(x-4)(x-2)}{(x-3)(x+2)}$

$\frac{(x-2)}{(x+5)(x-3)}$

50. $\frac{(x-5)(x-5)}{(x+2)(x+3)} \div \frac{x(5-x)}{x^2(x+2)}$

$\frac{x^2(x+2)}{x(5-x)}$

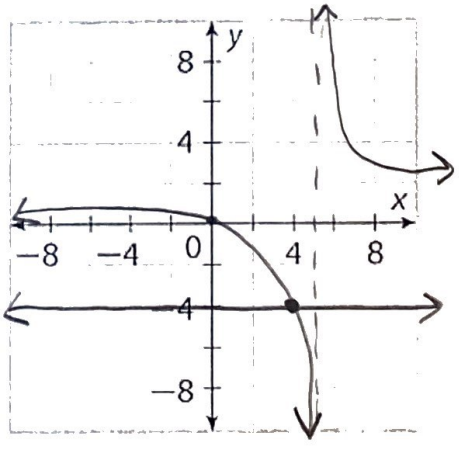
$-(x-5)$

$\frac{x(x-5)}{-(x+3)}$

51. Solve the rational equation by graphing.

$\frac{x}{x-5} = -4$

$x = 4$



Find the LCD of the following rational expressions.

52. $\frac{x^2-4}{5x-30}$ and $\frac{5x+13}{7x-42}$

$5(x-6)$ $7(x-6)$

$35(x-6)$

53. $\frac{x+4}{2x^2-32}$ and $\frac{x-6}{8(x-4)}$

$2(x^2-16)$ $8(x-4)$

$2(x+4)(x-4)$

$8(x+4)(x-4)$

Solve the rational expression.

54. $\frac{x+3}{(x+2)(x+1)} + \frac{1}{x+1} = \frac{3}{x+2}$ $\quad (x+2)$ $\quad (x+1)$

$x+3+x+2=3x+3$

$2x+5=3x+3$

$-3 \quad -2x$

$2=x$

$x=2$

55. $x \cdot x - \frac{36 \cdot x}{x} = 5 \cdot x$

$x^2-36=5x$

$x^2-5x-36=0$

$(x-9)(x+4)=0$

$x=9 \quad x=-4$

56. Working together, Mark and his sister Susan can clean the house in 3 hours. Working alone, it takes Susan 4 hours longer than Mark when he works alone. Find an equation that can be used to find the time, t, that it takes Mark to clean the house working alone.

$$\frac{1}{t} + \frac{1}{t+4} = \frac{1}{3}$$

Find the explicit **and** recursive rules for the sequences listed below.

57.

n	1	2	3	4	5
f(n)	8	12	16	20	24

E: $f(n)=8+4(n-1)$

R: $f(1)=8$

$f(n)=f(n-1)+4$ for $n \geq 2$

58.

n	0	1	2	3	4	...
f(n)	0.5	1.5	4.5	13.5	40.5	...

E: $f(n)=0.5(3)^n$

R: $f(0)=0.5$

$f(n)=3 \cdot f(n-1)$ for $n \geq 1$

59. Eric is selling items at a farmer's market. His first hour of sales is $50. His sales grow by 20% each hour. What are his sales after the first 4 hours?

hour	1	2	3	4
$	50	60	72	86.40

= $ 268.40

60. A new store has sales of $2000 its first week, and its sales grow by 2% each week after. What will the stores sales be in week 6?

1	2	
2000		

$2,000(1.02)^{n-1}$

$2,000(1.02)^{6-1}$

week 6 = $2,208.16

61. Given the recursive rule for an arithmetic sequence, write the explicit rule.

$f(0) = 7 \text{ and } f(n) = f(n-1) + 36 \text{ for } n \geq 1$

$f(n) = 7 + 36n$

62. Given the explicit rule for an arithmetic sequence, write the recursive rule.

$f(n) = 18 - 5(n-1)$

$f(1) = 18$

$f(n) = f(n-1) - 5 \quad \text{for } n \geq 2$

63. A baseball club is holding a board game tournament for a fundraiser and it begins with 243 players. There are three players that play in each game. Each game with only has one winner, and that one winner advances to the next round. Write an explicit rule and use it to find out how many rounds the winner will need to play?

1	2	3	4	5
81	27	9	3	1

$f(n) = 81 \cdot \frac{1}{3}^{n-1}$

5 rounds

64. An online video game tournament begins with 256 players. Two players play in each game. In each game, there is only one winner that advances to next round. How many games will the winner play?

$a = 128$

$r = \frac{1}{2}$

1	2	3	4	5	6	7	8
128	64	32	16	8	4	2	1

8 rounds